4 Quadrilaterals

Revision Exercise 4 (p. 4.60)

Level 1

1. $\because$ $ABCD$ is a parallelogram.

$\therefore$ $AD = BC$ (opp. sides of // gram)

$b - 1 = 2b - 5$

$b = \underline{\underline{4}}$

$\angle C = \angle A$ (opp. $\angle$s of // gram)

$x = \underline{\underline{135°}}$

$\angle A + \angle B = 180°$ (int. $\angle$s, AD // BC)

$135° + y = 180°$

$y = \underline{\underline{45°}}$

2. $\because$ $ABCD$ is a parallelogram.

$\therefore$ $2y - 2x = 2x + 2$ (diags. of // gram)

$2y = 4x + 2$

$y = 2x + 1$ .....(1)

$x + 1 = y - 1$ (diags. of // gram)

$y = x + 2$ .....(2)

By substituting (2) into (1), we have

$x + 2 = 2x + 1$

$x = \underline{\underline{1}}$

By substituting $x = 1$ into (2), we have

$y = 1 + 2$

$= \underline{\underline{3}}$

3. $\because$ $ABCD$ is a rectangle.

$\therefore$ $ED = EC$ (property of rectangle)

$\therefore$ $\angle EDC = \angle ECD$ (base $\angle$s, isos. $\triangle$)

$= b$

In $\triangle ECD$,

$b + b + 80° = 180°$ ($\angle$ sum of $\triangle$)

$b = \underline{\underline{50°}}$

In $\triangle ACD$,

$\angle ADC = 90°$ (property of rectangle)

$a + b + \angle ADC = 180°$ ($\angle$ sum of $\triangle$)

$a + 50° + 90° = 180°$

$a = \underline{\underline{40°}}$

4. $\because$ $ABCD$ is a rectangle.

$\therefore$ AD // BC (property of rectangle)

$\therefore$ $\angle BCE = y$ (alt. $\angle$s, BC // AD)

In $\triangle FBC$,

$y + 40° + 80° = 180°$ ($\angle$ sum of $\triangle$)

$y = \underline{\underline{60°}}$

$\angle BCE + \angle ECD = 90°$ (property of rectangle)

$60° + x = 90°$

$x = \underline{\underline{30°}}$

5. $\because$ $ABCD$ is a rhombus.

$\therefore$ $\angle AED = 90°$ (property of rhombus)

In $\triangle AED$,

$3x + 2x + 90° = 180°$ ($\angle$ sum of $\triangle$)

$5x = 90°$

$x = \underline{\underline{18°}}$

$y = 2x$ (property of rhombus)

$= 2 \times 18°$

$= \underline{\underline{36°}}$

$z = 3x$ (property of rhombus)

$= 3 \times 18°$

$= \underline{\underline{54°}}$

6. $\because$ $ABCD$ is a rhombus.

$\therefore$ $\angle BAC = \angle BCA = 54°$ (property of rhombus)

In $\triangle ABC$,

$x + 54° + 54° = 180°$ ($\angle$ sum of $\triangle$)

$x = \underline{\underline{72°}}$

7. $\because$ $ABCD$ is a kite with $AB = AD$ and $BC = CD$.

$\therefore$ $y = \underline{\underline{35°}}$

$x = \underline{\underline{12}}$

$\angle CED = 90°$

In $\triangle CDE$,

$CE^2 + DE^2 = CD^2$ (Pyth. theorem)

$CE = \sqrt{CD^2 - DE^2}$

$z = \sqrt{15^2 - 12^2}$

$= \underline{\underline{9}}$

8. In $\triangle ADG$,

$\because$ $AC = (4 + 4)\text{ cm} = 8\text{ cm} = CD$ and

$AF = (3 + 3)\text{ cm} = 6\text{ cm} = FG$

$\therefore$ $CF = \frac{1}{2} \times DG$ (mid-pt. theorem)

$y = \frac{1}{2} \times 18$

$= \underline{\underline{9}}$

$\because$ $AB = BC$ and $AE = EF$

$\therefore$ $BE = \frac{1}{2} \times CF$ (mid-pt. theorem)

$x = \frac{1}{2} \times 9$

$= \underline{\underline{4.5}}$

9. $\because$ AE // BF // CG and $AB = BC$

$\therefore$ $EF = FG$ (intercept theorem)

$x = y$

$\because$ BF // CG // DH and $BC = CD$

$\therefore$ $FG = GH$ (intercept theorem)

$y = z$

$\therefore$ $x = y = z$

$\because$ $x + y + z = 18$

$3x = 18$

$\therefore$ $x = y = z = \underline{\underline{6}}$

10. $\because$ AE // BF // CG and $AB = BC$

$\therefore$ $EF = FG$ (intercept theorem)

$2x + 3 = 4x - 5$

$2x = 8$

$x = \underline{\underline{4}}$

$FG = (4x - 5)$ cm
$= (4 \times 4 - 5)$ cm
$= 11$ cm
$\because$ $BF // CG // DH$ and $FG = 11$ cm $= GH$
$\therefore$ $CD = BC$ (intercept theorem)
$y = \underline{\underline{5}}$

11. In $\triangle AEC$,
$\because$ $AD = DC$ and $FD // EC$
$\therefore$ $EF = FA$ (intercept theorem)
$= 5$ cm
$EA = EF + FA$
$= (5 + 5)$ cm
$= 10$ cm
In $\triangle ABC$,
$\because$ $AD = DC$ and $ED // BC$
$\therefore$ $BE = EA$ (intercept theorem)
$= 10$ cm
$BA = BE + EA$
$x = 10 + 10$
$= \underline{\underline{20}}$

12. In $\triangle ACD$,
$\because$ $DF = FC$ and $AD // EF$
$\therefore$ $AE = EC$ (intercept theorem)
$y = \underline{\underline{2}}$
In $\triangle BCD$,
$\because$ $DF = FC$ and $EF // BC$
$\therefore$ $DE = EB$ (intercept theorem)
$x = \underline{\underline{3}}$

13. $\because$ $ABCD$ is a square.
$\therefore$ $AB = BC = CD = AD$ (property of square)
$\therefore$ $BC = AB$
$2a - 3 = 8$
$a = \underline{\underline{5.5}}$
$CD = AB$
$b = \underline{\underline{8}}$
$AD = AB$
$c + 1 = 8$
$c = \underline{\underline{7}}$
In $\triangle ABE$,
$\angle AEB = 90°$
and $BE = AE = d$ cm (property of square)
$AE^2 + BE^2 = AB^2$ (Pyth. theorem)
$d^2 + d^2 = 8^2$
$2d^2 = 64$
$d^2 = 32$
$d = \underline{\underline{\sqrt{32}}}$ (or $4\sqrt{2}$)

14. $\because$ $PQRS$ is a square.
$\therefore$ $\angle SQR = 45°$ (property of square)
In $\triangle BQS$,
$\angle SBQ + \angle BSQ = \angle SQR$ (ext. $\angle$ of $\triangle$)
$\angle SBQ + 15° = 45°$
$\angle SBQ = \underline{\underline{30°}}$

15. $\angle EAB = 60°$ (prop. of equil. $\triangle$)
$\angle BAD = 90°$ (property of square)
$\angle EAD = \angle EAB + \angle BAD$
$= 60° + 90°$
$= 150°$
$\because$ $\triangle AEB$ is an equilateral triangle.
$\therefore$ $EA = AB$
Also, $AB = AD$ (property of square)
$\therefore$ $EA = AD$
$\therefore$ $\angle ADE = \angle AED$ (base $\angle$s, isos. $\triangle$)
In $\triangle AED$,
$\angle EAD + \angle ADE + \angle AED = 180°$ ($\angle$ sum of $\triangle$)
$150° + 2\angle ADE = 180°$
$\angle ADE = 15°$
$\angle ADB = 45°$ (property of square)
$\angle EDB = \angle ADB - \angle ADE$
$= 45° - 15°$
$= \underline{\underline{30°}}$

16. $\because$ $AQDR$ is a rectangle.
$\therefore$ $\angle R = 90°$, $DQ = AR = 3$ cm and
$AQ = DR = 4$ cm (property of rectangle)
In $\triangle ADR$,
$AD^2 = AR^2 + DR^2$ (Pyth. theorem)
$AD = \sqrt{AR^2 + DR^2}$
$= \sqrt{3^2 + 4^2}$ cm
$= 5$ cm
$\because$ $ABCD$ is a rhombus.
$\therefore$ $AB = CD = AD = 5$ cm,
$CQ = AQ = 4$ cm and
$BQ = DQ = 3$ cm (property of rhombus)
$\because$ $BPCQ$ is a rectangle.
$\therefore$ $CP = BQ = 3$ cm and
$BP = CQ = 4$ cm (property of rectangle)
Perimeter of polygon $ABPCDR$
$= AB + BP + CP + CD + DR + AR$
$= (5 + 4 + 3 + 5 + 4 + 3)$ cm
$= \underline{\underline{24 \text{ cm}}}$

17. **(a)** $\because$ Sum of the interior angles of a quadrilateral is 360°.
$\therefore$ $\angle A + \angle B + \angle C + \angle D = 360°$
$4a + 8° + 5a - 35° + 100° + 2a + 34° = 360°$
$11a + 107° = 360°$
$11a = 253°$
$a = \underline{\underline{23°}}$

(b) $\angle A = 4 \times 23° + 8°$
$= 100°$
$\angle B = 5 \times 23° - 35°$
$= 80°$
$\angle C = 100°$ given
$\angle D = 2 \times 23° + 34°$
$= 80°$
$\therefore$ $\angle A = \angle C$ and $\angle B = \angle D$
$\therefore$ $ABCD$ is a parallelogram. opp. $\angle$s equal

(c) $\because$ $ABCD$ is a parallelogram. (proved in (b))

$\therefore \quad AD = BC$ (opp. sides of // gram)

$6 + 2b = 4b - 12$

$2b = 18$

$b = \underline{\underline{9}}$

18. **(a)** In $\triangle ABP$ and $\triangle CDQ$,

$\angle ABP = \angle CDQ$	alt. $\angle$s, AB // DC
$\because \quad \angle APQ = \angle CQP$	given
$\therefore \quad \angle APB = 180° - \angle APQ$	adj. $\angle$s on st. line
$= 180° - \angle CQP$	
$= \angle CQD$	adj. $\angle$s on st. line
$AB = CD$	opp. sides of // gram
$\therefore \quad \triangle ABP \cong \triangle CDQ$	AAS

(b)

$\because \quad \angle APQ = \angle CQP$	given
$\therefore \quad AP // CQ$	alt. $\angle$s equal
$\because \quad \triangle ABP \cong \triangle CDQ$	proved in (a)
$\therefore \quad AP = CQ$	corr. sides, $\cong \triangle$s
$\therefore$ $APCQ$ is a parallelogram.	opp. sides equal and //

19. $\because$ $\triangle CDE$ is an equilateral triangle.

$\therefore \quad \angle CED = \angle ECD = \angle EDC = 60°$ (prop. of equil. $\triangle$)

$\because$ $ABCE$ is a rhombus.

$\therefore \quad \angle AEC = \angle ABC$ (property of rhombus)

$= 100°$

$\angle AED = \angle AEC + \angle CED$

$= 100° + 60°$

$= 160°$

$\because \quad EC // AB$ (property of rhombus)

$\therefore \quad \angle BCE = 180° - \angle ABC$ (int. $\angle$s, EC // AB)

$= 80°$

$\angle BCD = \angle BCE + \angle ECD$

$= 80° + 60°$

$= 140°$

$\because$ $\triangle CDE$ is an equilateral triangle.

$\therefore \quad EC = CD = DE$

Also, $AB = BC = CE = EA$ (property of rhombus)

$\therefore \quad DE = EA$ and $CD = CB$

$\therefore \quad \angle EDA = \angle EAD$ and

$\angle CDB = \angle CBD$ (base $\angle$s, isos. $\triangle$)

In $\triangle AED$,

$\angle AED + \angle EDA + \angle EAD = 180°$ ($\angle$ sum of $\triangle$)

$160° + 2\angle EDA = 180°$

$\angle EDA = 10°$

In $\triangle BCD$,

$\angle BCD + \angle CBD + \angle CDB = 180°$ ($\angle$ sum of $\triangle$)

$140° + 2\angle CDB = 180°$

$\angle CDB = 20°$

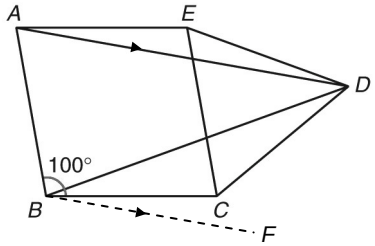
$\angle ADB = \angle EDC - \angle EDA - \angle CDB$

$= 60° - 10° - 20°$

$= \underline{\underline{30°}}$

Alternative Solution

Construct BF such that BF // AD.



Let $\angle EAD = a$ and $\angle CBD = b$.

$\because \quad AE // BC$ (property of rhombus)

$\therefore \quad a + \angle DAB + 100° = 180°$ (int. $\angle$s, AE // BC)

$\angle DAB = 80° - a$

$\angle DAB + \angle ABF = 180°$ (int. $\angle$s, AD // BF)

$(80° - a) + (100° + \angle CBF) = 180°$

$\angle CBF = a$

$\angle ADB = \angle CBD + \angle CBF$ (alt. $\angle$s, AD // BF)

$= b + a$

$\because$ $\triangle CDE$ is an equilateral triangle.

$\therefore \quad EC = CD = DE$

Also, $AB = BC = CE = EA$ (property of rhombus)

$\therefore \quad DE = EA$ and $CD = CB$

$\therefore \quad \angle EDA = \angle EAD = a$ and

$\angle CDB = \angle CBD = b$ (base $\angle$s, isos. $\triangle$)

$\because \quad \angle CDE = 60°$ (prop. of equil. $\triangle$)

$\therefore \quad \angle EDA + \angle ADB + \angle CDB = 60°$

$a + (b + a) + b = 60°$

$b + a = 30°$

$\therefore \quad \angle ADB = b + a$

$= \underline{\underline{30°}}$

20. $\because$ $ABCD$ is a square.

$\therefore \quad \angle DBC = 45°$ (property of square)

$\because \quad BF // CE$ (property of rhombus)

$\therefore \quad \angle BCE = 180° - \angle DBC$ (int. $\angle$s, BF // CE)

$= 180° - 45°$

$= 135°$

$\because \quad AB = BC = CD = DA$ (property of square)

$DC = CE = EF = FD$ (property of rhombus)

$\therefore \quad BC = CE$

$\therefore \quad \angle EBC = \angle BEC$ (base $\angle$s, isos. $\triangle$)

In $\triangle BCE$,

$\angle BCE + \angle EBC + \angle BEC = 180°$ ($\angle$ sum of $\triangle$)

$135° + 2\angle EBC = 180°$

$\angle EBC = 22.5°$

$\angle DBG = \angle DBC - \angle EBC$

$= 45° - 22.5°$

$= \underline{\underline{22.5°}}$

21. In $\triangle OAB$,

$\because \quad OD = DA$ and $ED // BA$

$\therefore \quad OE = EB$ (intercept theorem)

$\because \quad OE = EB$ and $OD = DA$

$\therefore \quad ED = \frac{1}{2} \times AB$ (mid-pt. theorem)

$= \frac{1}{2} \times 10 \text{ cm}$

$= 5 \text{ cm}$

In $\triangle OCA$,

$\because$ $OD = DA$ and $FD // CA$

$\therefore$ $OF = FC$ (intercept theorem)

$\because$ $OF = FC$ and $OD = DA$

$\therefore$ $FD = \frac{1}{2} \times CA$ (mid-pt. theorem)

$= \frac{1}{2} \times 8\text{ cm}$

$= 4\text{ cm}$

In $\triangle OBC$,

$\because$ $OE = EB$ and $OF = FC$

$\therefore$ $EF = \frac{1}{2} \times BC$ (mid-pt. theorem)

$= \frac{1}{2} \times 6\text{ cm}$

$= 3\text{ cm}$

Perimeter of $\triangle DEF = (5 + 4 + 3)\text{ cm}$

$= \underline{\underline{12\text{ cm}}}$

22. **(a)** Let $\angle CBE = a$ and $\angle BCE = b$.

$\angle ABE = \angle CBE = a$	given
$\angle DCE = \angle BCE = b$	given
$\angle ABC + \angle BCD = 180°$	int. $\angle$s, $AB // DC$
$2a + 2b = 180°$	
$a + b = 90°$	
In $\triangle BCE$,	
$\angle BEC + \angle BCE + \angle CBE = 180°$	$\angle$ sum of $\triangle$
$\angle BEC + a + b = 180°$	
$\angle BEC + 90° = 180°$	
$\angle BEC = 90°$	
$\therefore$ $\triangle BEC$ is a right-angled triangle.	

(b) In $\triangle BCE$,

$BC^2 = BE^2 + CE^2$ (Pyth. theorem)

$BC = \sqrt{BE^2 + CE^2}$

$= \sqrt{12^2 + 9^2}\text{ cm}$

$= 15\text{ cm}$

$AD = BC = 15\text{ cm}$ (opp. sides of // gram)

$\because$ $\angle DEC = \angle BCE$ (alt. $\angle$s, $DE // CB$)

and $\angle DCE = \angle BCE$

$\therefore$ $\angle DCE = \angle DEC$

$\therefore$ $DC = DE$ (sides opp. equal $\angle$s)

$= 7.5\text{ cm}$

$AB = DC = 7.5\text{ cm}$ (opp. sides of // gram)

Perimeter of parallelogram $ABCD$

$= AB + BC + CD + AD$

$= (7.5 + 15 + 15 + 7.5)\text{ cm}$

$= \underline{\underline{45\text{ cm}}}$

Level 2

23. $\because$ $ABCD$ is a rhombus.

$\therefore$ $\angle ADB = \angle CBD = \angle ABD$ (property of rhombus)

$\therefore$ $\angle ADB = \angle ABD$

$= \frac{1}{2} \times \angle ABC$

$= \frac{1}{2} \times 72°$

$= 36°$

$\angle BDE = 180° - \angle ADB$ (adj. $\angle$s on st. line)

$= 180° - 36°$

$= 144°$

$\angle BDF = \frac{1}{2} \times \angle BDE$ (given)

$= \frac{1}{2} \times 144°$

$= 72°$

$\angle DOA = 90°$ (property of rhombus)

$\angle AFD = \angle DOA - \angle BDF$ (ext. $\angle$ of $\triangle$)

$= 90° - 72°$

$= \underline{\underline{18°}}$

24. $\because$ $\triangle BCG$ is an equilateral triangle.

$\therefore$ $CG = GB = BC$

Also, $BC = CD$ (property of square)

$\therefore$ $CD = CG$

$\therefore$ $\angle CDG = \angle CGD$ (base $\angle$s, isos. $\triangle$)

$\because$ $\angle BCG = 60°$ (prop. of equil. $\triangle$)

$\angle DCB = 90°$ (property of square)

$\therefore$ $\angle DCG = \angle DCB - \angle BCG$

$= 90° - 60°$

$= 30°$

In $\triangle CDG$,

$\angle DCG + \angle CDG + \angle CGD = 180°$ ($\angle$ sum of $\triangle$)

$30° + 2\angle CDG = 180°$

$\angle CDG = 75°$

$\angle CDB = 45°$ (property of square)

$\angle GDB = \angle CDG - \angle CDB$

$= 75° - 45°$

$= \underline{\underline{30°}}$

25. In $\triangle ADQ$ and $\triangle CBP$,

$\angle ADB = \angle CBD$	alt. $\angle$s, $AD // BC$
$\angle ADQ = 180° - \angle ADB$	adj. $\angle$s on st. line
$= 180° - \angle CBD$	
$= \angle CBP$	adj. $\angle$s on st. line
$AD = CB$	opp. sides of // gram
$DQ = BP$	given
$\therefore$ $\triangle ADQ \cong \triangle CBP$	SAS
$\therefore$ $AQ = CP$	corr. sides, $\cong \triangle$s
$\angle AQD = \angle CPB$	corr. $\angle$s, $\cong \triangle$s
$\therefore$ $AQ // CP$	alt. $\angle$s equal
$\therefore$ $APCQ$ is a parallelogram.	opp. sides equal and //

26. **(a)** $\because$ $KB = KC$ (property of rectangle)

$\therefore$ $\angle KBC = \angle KCB$ (base $\angle$s, isos. $\triangle$)

$= 24°$

$\angle BKC = 180° - \angle KBC - \angle KCB$ ($\angle$ sum of $\triangle$)

$= 180° - 24° - 24°$

$= 132°$

$\angle AKD = \angle BKC = 132°$ (vert. opp. $\angle$s)
$\angle AKE = 60°$ (prop. of equil. $\triangle$)
$\angle EKD = \angle AKD - \angle AKE$
$= 132° - 60°$
$= \underline{\underline{72°}}$

(b) $\because$ $\triangle AKE$ is an equilateral triangle.
$\therefore$ $KA = KE$
Also, $KA = KD$ (property of rectangle)
$\therefore$ $KE = KD$
$\therefore$ $\angle KED = \angle KDE$ (base $\angle$s, isos. $\triangle$)
$\angle EKD + \angle KDE + \angle KED = 180°$ ($\angle$ sum of $\triangle$)
$72° + 2\angle KDE = 180°$
$\angle KDE = 54°$
$\angle KDA = \angle KBC$ (alt. $\angle$s, AD // BC)
$= 24°$
$\angle EDA = \angle KDE - \angle KDA$
$= 54° - 24°$
$= \underline{\underline{30°}}$

27. (a) In $\triangle BCE$ and $\triangle BGA$,
$\angle GBE = \angle CBA = 90°$ — property of rectangle
$\angle CBE = \angle CBG + \angle GBE$
$= \angle CBG + \angle CBA$
$= \angle GBA$
$\because$ $ABCD$ and $BEFG$ are two congruent rectangles.
$\therefore$ $BA = BE$ and $BG = BC$
$\therefore$ $\triangle BCE \cong \triangle BGA$ — SAS

(b) $\because$ $\triangle BCE \cong \triangle BGA$ — proved in (a)
$\therefore$ $\angle BCE = \angle BGA$ — corr. $\angle$s, $\cong$ $\triangle$s
$\angle BPG = 180° - \angle PBQ - \angle BGA$ — $\angle$ sum of $\triangle$
$= 180° - \angle PBQ - \angle BCE$
$= \angle BQC$ — $\angle$ sum of $\triangle$

28. (a) (i) In $\triangle OCD$ and $\triangle OQP$,
$\angle COD = \angle QOP$ — common angle
$\because$ $CO = DO$ — property of rectangle
and $CQ = DP$ — given
$\therefore$ $QO = CO + CQ$
$= DO + DP$
$= PO$
$\therefore$ $\dfrac{CO}{QO} = \dfrac{DO}{PO}$
$\therefore$ $\triangle OCD \sim \triangle OQP$ — ratio of 2 sides, inc. $\angle$
$\therefore$ $\angle OCD = \angle OQP$ — corr. $\angle$s, $\sim$ $\triangle$s
$\therefore$ CD // PQ — corr. $\angle$s equal
Also, AB // CD — property of rectangle
$\therefore$ AB // PQ

(ii) In $\triangle AOP$ and $\triangle BOQ$,
$OP = OQ$ — proved in (a)
$\angle AOP = \angle BOQ$ — vert. opp. $\angle$s
$AO = BO$ — property of rectangle
$\therefore$ $\triangle AOP \cong \triangle BOQ$ — SAS
$\therefore$ $AP = BQ$ — corr. sides, $\cong$ $\triangle$s

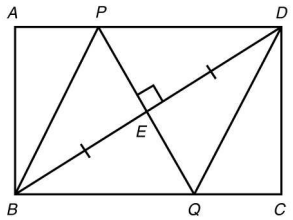
(b) $\because$ $ABQP$ is a quadrilateral with AB // PQ and $AP = BQ$.
$\therefore$ $ABQP$ is an isosceles trapezium.

29. (a) In $\triangle ABC$ and $\triangle AFE$,
$\angle BAC = \angle FAE$ — common angle
$\angle ABC = 90°$ — property of square
$= \angle AFE$ — given
$AC = AE$ — given
$\therefore$ $\triangle ABC \cong \triangle AFE$ — AAS

(b) In $\triangle ABG$ and $\triangle AFG$,
$\angle ABG = \angle AFG = 90°$ — proved in (a)
$\because$ $\triangle ABC \cong \triangle AFE$ — proved in (a)
$\therefore$ $AB = AF$ — corr. sides, $\cong$ $\triangle$s
$AG = AG$ — common side
$\therefore$ $\triangle ABG \cong \triangle AFG$ — RHS

(c) $\angle BAC = 45°$ (property of square)
$\because$ $\triangle ABG \cong \triangle AFG$ (proved in (b))
$\therefore$ $\angle BAG = \angle FAG$ (corr. $\angle$s, $\cong$ $\triangle$s)
$= \dfrac{1}{2} \times \angle BAC$
$= \dfrac{1}{2} \times 45°$
$= 22.5°$
In $\triangle ABG$,
$\angle AGB + \angle BAG + \angle ABG = 180°$ ($\angle$ sum of $\triangle$)
$\angle AGB + 22.5° + 90° = 180°$
$\angle AGB = \underline{\underline{67.5°}}$

30. With the notations in the figure,



in $\triangle PED$ and $\triangle QEB$,
$\because$ PD // BQ — property of rectangle
$\therefore$ $\angle DPE = \angle BQE$ — alt. $\angle$s, PD // BQ
$\angle PED = \angle QEB$ — vert. opp. $\angle$s
$ED = EB$ — given
$\therefore$ $\triangle PED \cong \triangle QEB$ — AAS
$\therefore$ $PE = QE$ — corr. sides, $\cong$ $\triangle$s
and $ED = EB$ — given
$\therefore$ $BQDP$ is a parallelogram. — diags. bisect each other
In $\triangle PED$ and $\triangle PEB$,
$\angle PED = \angle PEB = 90°$ — given
$ED = EB$ — given
$PE = PE$ — common side
$\therefore$ $\triangle PED \cong \triangle PEB$ — SAS
$\therefore$ $PD = PB$ — corr. sides, $\cong$ $\triangle$s
Also, $PD = BQ$ and $PB = QD$ — opp. sides of // gram
$\therefore$ $PD = QD = BQ = PB$
$\therefore$ $BQDP$ is a parallelogram with four equal sides.
$\therefore$ $BQDP$ is a rhombus.

31. (a) In $\triangle APR$ and $\triangle CQR$,

$AR = CR$	given
$\angle ARP = \angle CRQ$	vert. opp. $\angle$s
$\angle PAR = \angle QCR$	alt. $\angle$s, $PA \parallel CQ$
$\therefore \triangle APR \cong \triangle CQR$	ASA

(b)

$\because \triangle APR \cong \triangle CQR$	proved in (a)
$\therefore PA = QC$	corr. sides, $\cong \triangle$s
and $BQ = QC$	given
$\therefore PA = BQ$	
Also, $PA \parallel QB$	given
$\therefore ABQP$ is a parallelogram.	opp. sides equal and $\parallel$

(c) In $\triangle ADE$,

$\because AB = BD$ and $AC = CE$	given
$\therefore BC = \frac{1}{2} \times DE$	mid-pt. theorem
$DE = 2BC$	
$\because BQ = QC$	given
$\therefore BC = 2BQ$	
$= 2AP$	proved in (a)
$\therefore DE = 2BC = 4AP$	

32. In $\triangle ABC$,

$\because AE = EB$ and $EH \parallel BC$	
$\therefore AH = HC$	intercept theorem
$\because AE = EB$ and $AH = HC$	
$\therefore EH = \frac{1}{2} \times BC$	mid-pt. theorem
$EG + GH = \frac{1}{2} \times BC$	
$GH = \frac{1}{2} \times BC - EG$	

In $\triangle ADB$,

$\because AE = EB$ and $EG \parallel AD$	
$\therefore DG = GB$	intercept theorem
$\because AE = EB$ and $DG = GB$	
$\therefore EG = \frac{1}{2} \times AD$	mid-pt. theorem
$\therefore GH = \frac{1}{2} \times BC - \frac{1}{2} \times AD$	
$= \frac{1}{2} \times (BC - AD)$	

33. (a) In $\triangle AQB$ and $\triangle CPD$,

$\because \angle DAB = \angle QBA$	alt. $\angle$s, $DA \parallel BQ$
$\angle PDC = \angle DAB$	corr. $\angle$s, $DC \parallel AB$
$\therefore \angle QBA = \angle PDC$	
$AB = CD$	opp. sides of $\parallel$ gram
$QB = PD$	given
$\therefore \triangle AQB \cong \triangle CPD$	SAS

(b) In $\triangle ADF$ and $\triangle CBE$,

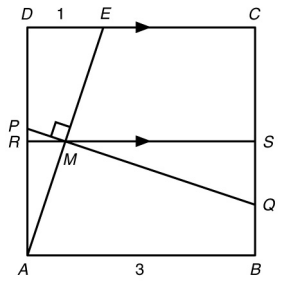
$\because \angle DCB = \angle EBC$	alt. $\angle$s, $DC \parallel BE$
$\angle FDA = \angle DCB$	corr. $\angle$s, $DA \parallel CB$
$\therefore \angle FDA = \angle EBC$	
$\because \triangle AQB \cong \triangle CPD$	proved in (a)
$\therefore \angle QAB = \angle PCD$	corr. $\angle$s, $\cong \triangle$s
Also, $\angle CEB = \angle PCD$	corr. $\angle$s, $BE \parallel DC$
$\angle AFD = \angle QAB$	corr. $\angle$s, $FD \parallel AB$
$\therefore \angle AFD = \angle CEB$	
$AD = CB$	opp. sides of $\parallel$ gram
$\therefore \triangle ADF \cong \triangle CBE$	AAS

(c)

$\because AD = BC$	opp. sides of $\parallel$ gram
$DP = QB$	given
$\therefore AP = AD + DP$	
$= BC + QB$	
$= QC$	
$AP \parallel QC$	
$\therefore AQCP$ is a parallelogram.	opp. sides equal and $\parallel$
$\because \triangle ADF \cong \triangle CBE$	proved in (b)
$\therefore FD = BE$	corr. sides, $\cong \triangle$s
Also, $DC = AB$	opp. sides of $\parallel$ gram
$\therefore FC = FD + DC$	
$= AB + BE$	
$= AE$	
$FC \parallel AE$	
$\therefore AECF$ is a parallelogram.	opp. sides equal and $\parallel$

Challenging Questions (p. 4.64)

1. Construct RM such that R is a point on DA and $RM \parallel DC$. RM produced meets CB at S.



$\because AM = ME$	(given)
$RM \parallel DE$	(by construction)
$\therefore AR = RD$	(intercept theorem)
$\because AR = RD$ and $AM = ME$	
$\therefore RM = \frac{1}{2} \times DE$	(mid-pt. theorem)
$= \frac{1}{2} \times 1$	
$= \frac{1}{2}$	
$\because RS = AB = 3$	
$\therefore MS = 3 - RM$	
$= 3 - \frac{1}{2}$	
$= \frac{5}{2}$	

In $\triangle MPR$ and $\triangle MQS$,

$\angle PMR = \angle QMS$	(vert. opp. $\angle$s)
$\because PR \parallel SQ$	(property of square)
$\therefore \angle PRM = \angle QSM$	(alt. $\angle$s, $PR \parallel SQ$)
$\angle RPM = \angle SQM$	(alt. $\angle$s, $PR \parallel SQ$)
$\therefore \triangle MPR \sim \triangle MQS$	(AAA)

$\therefore \quad \dfrac{PM}{QM} = \dfrac{RM}{SM}$ (corr. sides, ~ $\triangle$s)

$= \dfrac{\frac{1}{2}}{\frac{5}{2}}$

$= \dfrac{1}{5}$

$\therefore \quad \dfrac{PM}{MQ} = \dfrac{1}{5}$

2. In $\triangle BPM$ and $\triangle CPB$,

$\angle BPM = \angle CPB = 90°$	given
$\angle MBP + \angle PMB + 90° = 180°$	$\angle$ sum of $\triangle$
$\angle PMB = 90° - \angle MBP$	
$\angle MBP + \angle PBC = 90°$	property of square
$\angle PBC = 90° - \angle MBP$	
$\therefore \quad \angle PMB = \angle PBC$	
$\angle PBM = 180° - \angle BPM - \angle PMB$	$\angle$ sum of $\triangle$
$= 180° - \angle CPB - \angle PBC$	
$= \angle PCB$	$\angle$ sum of $\triangle$
$\therefore \quad \triangle BPM \sim \triangle CPB$	AAA
$\therefore \quad \dfrac{BP}{CP} = \dfrac{BM}{CB}$	corr. sides, ~ $\triangle$s
$\because \quad BM = BN$	given
and $CB = CD$	property of square
$\therefore \quad \dfrac{BP}{PC} = \dfrac{BN}{CD}$	
$\angle PBN = 90° - \angle MBP$	property of square
$= 90° - \angle BCP$	
$= \angle PCD$	
$\therefore \quad \triangle PBN \sim \triangle PCD$	ratio of 2 sides, inc. $\angle$
$\therefore \quad \angle BPN = \angle CPD$	corr. $\angle$s, ~ $\triangle$s
$\therefore \quad \angle DPN = \angle CPN + \angle CPD$	
$= \angle CPN + \angle BPN$	
$= 90°$	
i.e. $PD \perp PN$	