9 Trigonometric Relations

Review Exercise 9 (p. 9.3)

1. **(a)** $\tan 15° + \cos 55° = \underline{\underline{0.842}}$ (cor. to 3 sig. fig.)

(b) $\sin 24° - 4\tan 10° = \underline{\underline{-0.299}}$ (cor. to 3 sig. fig.)

(c) $\dfrac{\sin 34°}{\cos 68°} = \underline{\underline{1.49}}$ (cor. to 3 sig. fig.)

2. **(a)** $\sin\theta = 0.891$

$\theta = \underline{\underline{63.0°}}$ (cor. to the nearest 0.1°)

(b) $\tan\theta = \sin 66° \cos 22°$

$\tan\theta \approx 0.847\,025$

$\theta = \underline{\underline{40.3°}}$ (cor. to the nearest 0.1°)

(c) $\cos\theta = (\sin 78°)^2$

$\cos\theta \approx 0.956\,773$

$\theta = 16.9°$ (cor. to the nearest 0.1°)

3. **(a)** By Pythagoras' theorem,

$$\text{hypotenuse} = \sqrt{\text{opposite side}^2 + \text{adjacent side}^2}$$
$$= \sqrt{3^2 + 4^2}$$
$$= 5$$

$\therefore \quad \sin\theta = \dfrac{3}{5}, \cos\theta = \dfrac{4}{5}, \tan\theta = \dfrac{3}{4}$

(b) By Pythagoras' theorem,

$$\text{opposite side} = \sqrt{\text{hypotenuse}^2 - \text{adjacent side}^2}$$
$$= \sqrt{26^2 - 24^2}$$
$$= 10$$

$\therefore \quad \sin\theta = \dfrac{10}{26}, \cos\theta = \dfrac{24}{26}, \tan\theta = \dfrac{10}{24}$

i.e. $\sin\theta = \dfrac{5}{13}, \cos\theta = \dfrac{12}{13}, \tan\theta = \dfrac{5}{12}$

(c) By Pythagoras' theorem,

$$\text{adjacent side} = \sqrt{\text{hypotenuse}^2 - \text{opposite side}^2}$$
$$= \sqrt{3^2 - 2^2}$$
$$= \sqrt{5}$$

$\therefore \quad \sin\theta = \dfrac{2}{3}, \cos\theta = \dfrac{\sqrt{5}}{3}, \tan\theta = \dfrac{2}{\sqrt{5}} \left(\text{or } \dfrac{2\sqrt{5}}{5}\right)$

4. **(a)** $\tan\theta = \dfrac{4}{2}$

$\theta = \underline{\underline{63.43°}}$ (cor. to 4 sig. fig.)

(b) $\cos 35° = \dfrac{r}{5}$

$r = 5\cos 35°$

$r = \underline{\underline{4.096}}$ (cor. to 4 sig. fig.)

(c) $\sin 25° = \dfrac{3}{x}$

$x = \dfrac{3}{\sin 25°}$

$x = \underline{\underline{7.099}}$ (cor. to 4 sig. fig.)

Activity

Activity 9.1 (p. 9.4)

1. **(a)** **(i)** $AC = \sqrt{1^2 + 1^2} = \sqrt{2}$

(ii) $x = 45°$

(b) $\sin 45° = \dfrac{1}{\sqrt{2}}\left(\text{or } \dfrac{\sqrt{2}}{2}\right)$, $\cos 45° = \dfrac{1}{\sqrt{2}}\left(\text{or } \dfrac{\sqrt{2}}{2}\right)$,

$\tan 45° = 1$

2. **(a)** **(i)** $PS = 1$

$RS = \sqrt{2^2 - 1^2} = \sqrt{3}$

(ii) $\theta = 30°$

(b) **(i)** $\sin 60° = \dfrac{\sqrt{3}}{2}$, $\cos 60° = \dfrac{1}{2}$, $\tan 60° = \sqrt{3}$

(ii) $\sin 30° = \dfrac{1}{2}$, $\cos 30° = \dfrac{\sqrt{3}}{2}$,

$\tan 30° = \dfrac{1}{\sqrt{3}}\left(\text{or } \dfrac{\sqrt{3}}{3}\right)$

Activity 9.2 (p. 9.15)

1.

θ	$\dfrac{\sin\theta}{\cos\theta}$	$\sin^2\theta + \cos^2\theta$
30°	$\underline{\dfrac{1}{\sqrt{3}}}$	$\underline{1}$
45°	$\underline{1}$	$\underline{1}$
60°	$\underline{\sqrt{3}}$	$\underline{1}$

2. **(a)** yes

(b) $\sin\theta = \dfrac{b}{c}$, $\cos\theta = \dfrac{a}{c}$, $\tan\theta = \dfrac{b}{a}$

(i) $\dfrac{\sin\theta}{\cos\theta} = \dfrac{\left(\dfrac{b}{c}\right)}{\left(\dfrac{a}{c}\right)} = \dfrac{b}{a} = \tan\theta$

(ii) $c^2 = a^2 + b^2$

$$\sin^2\theta + \cos^2\theta = \left(\frac{b}{c}\right)^2 + \left(\frac{a}{c}\right)^2$$
$$= \frac{b^2 + a^2}{c^2}$$
$$= 1$$

Activity 9.3 (p. 9.23)

1. $\sin\theta = \frac{a}{b}$, $\sin(90° - \theta) = \frac{c}{b}$,

$\cos\theta = \frac{c}{b}$, $\cos(90° - \theta) = \frac{a}{b}$,

$\tan\theta = \frac{a}{c}$, $\tan(90° - \theta) = \frac{c}{a}$

2. **(a)** $\sin(90° - \theta) = \cos\theta$

(b) $\cos(90° - \theta) = \sin\theta$

(c) $\tan(90° - \theta) = \frac{1}{\tan\theta}$

Quick Practice

Quick Practice 9.1 (p. 9.6)

(a) $\sin 30° - \tan 45° \cos 60° = \frac{1}{2} - 1 \times \frac{1}{2}$

$= \frac{1}{2} - \frac{1}{2}$

$= \underline{\underline{0}}$

(b) $\frac{\cos^2 30°}{\tan 30° \sin 60°} = \frac{\left(\frac{\sqrt{3}}{2}\right)^2}{\frac{1}{\sqrt{3}} \times \frac{\sqrt{3}}{2}}$

$= \frac{\frac{3}{4}}{\frac{1}{2}}$

$= \underline{\underline{\frac{3}{2}}}$

(c) $\tan^2 60° + \sin^2 45° = \sqrt{3}^2 + \left(\frac{1}{\sqrt{2}}\right)^2$

$= 3 + \frac{1}{2}$

$= \underline{\underline{\frac{7}{2}}}$

Quick Practice 9.2 (p. 9.7)

(a) $3\tan\theta - 2\cos 30° = 0$

$3\tan\theta - 2 \times \frac{\sqrt{3}}{2} = 0$

$3\tan\theta = \sqrt{3}$

$\tan\theta = \frac{\sqrt{3}}{3}$

$\theta = \underline{\underline{30°}}$

(b) $2\sin\frac{\theta}{2} = 1$

$\sin\frac{\theta}{2} = \frac{1}{2}$

$\frac{\theta}{2} = 30°$

$\theta = \underline{\underline{60°}}$

Quick Practice 9.3 (p. 9.8)

Consider $\triangle ABC$.

$\sin 45° = \frac{AB}{AC}$

$\frac{1}{\sqrt{2}} = \frac{10}{x}$

$x = \underline{\underline{10\sqrt{2}}}$

Consider $\triangle ACD$.

$\tan 30° = \frac{CD}{AC}$

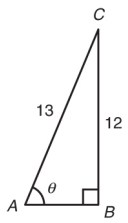
$\frac{1}{\sqrt{3}} = \frac{y}{10\sqrt{2}}$

$y = 10\sqrt{2} \times \frac{1}{\sqrt{3}}$

$= \underline{\underline{\frac{10\sqrt{2}}{\sqrt{3}}}} \left(\text{or } \frac{10\sqrt{6}}{3}\right)$

Quick Practice 9.4 (p. 9.12)

Construct $\triangle ABC$ as shown with $\sin\theta = \frac{12}{13}$.



By Pythagoras' theorem,

$AB = \sqrt{AC^2 - BC^2}$

$= \sqrt{13^2 - 12^2}$

$= 5$

By definition,

$\cos\theta = \frac{AB}{AC} = \underline{\underline{\frac{5}{13}}}$

$\tan\theta = \frac{BC}{AB} = \underline{\underline{\frac{12}{5}}}$

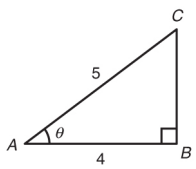
Quick Practice 9.5 (p. 9.13)

$\cos\theta = 0.8$

$= \frac{8}{10}$

$= \frac{4}{5}$

Construct $\triangle ABC$ as shown with $\cos\theta = \frac{4}{5}$.



By Pythagoras' theorem,

$BC = \sqrt{AC^2 - AB^2}$

$= \sqrt{5^2 - 4^2}$

$= 3$

By definition,

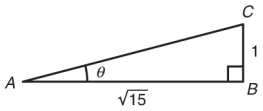
$\sin\theta = \frac{BC}{AC} = \underline{\underline{\frac{3}{5}}}$

$\tan\theta = \frac{BC}{AB} = \underline{\underline{\frac{3}{4}}}$

Quick Practice 9.6 (p. 9.13)

Construct $\triangle ABC$ as shown with $\tan\theta = \frac{1}{\sqrt{15}}$.

By Pythagoras' theorem,



$$AC = \sqrt{AB^2 + BC^2}$$
$$= \sqrt{(\sqrt{15})^2 + 1^2}$$
$$= 4$$

By definition,

$$\sin\theta = \frac{BC}{AC} = \frac{1}{4}$$
$$\cos\theta = \frac{AB}{AC} = \frac{\sqrt{15}}{4}$$
$$\sin\theta\cos\theta = \frac{1}{4}\times\frac{\sqrt{15}}{4}$$
$$= \frac{\sqrt{15}}{16}$$

Quick Practice 9.7 (p. 9.17)

(a) $$\frac{2\tan\theta}{\sin\theta} = \frac{2\left(\frac{\sin\theta}{\cos\theta}\right)}{\sin\theta}$$
$$= \frac{2\sin\theta}{\cos\theta}\times\frac{1}{\sin\theta}$$
$$= \frac{2}{\cos\theta}$$

(b) $$\sin^2\theta - (3-\cos^2\theta) = \sin^2\theta - 3 + \cos^2\theta$$
$$= (\sin^2\theta + \cos^2\theta) - 3$$
$$= 1 - 3$$
$$= -2$$

(c) $$\frac{\cos^2\theta}{2-2\sin^2\theta} = \frac{\cos^2\theta}{2(1-\sin^2\theta)}$$
$$= \frac{\cos^2\theta}{2\cos^2\theta}$$
$$= \frac{1}{2}$$

Quick Practice 9.8 (p. 9.18)

(a) $$(1+\tan^2\theta)\cos^2\theta = \left(1+\frac{\sin^2\theta}{\cos^2\theta}\right)\cos^2\theta$$
$$= \cos^2\theta + \sin^2\theta$$
$$= 1$$

(b) $$\frac{\cos\theta}{\tan\theta} + \sin\theta = \frac{\cos\theta}{\frac{\sin\theta}{\cos\theta}} + \sin\theta$$
$$= \frac{\cos^2\theta}{\sin\theta} + \frac{\sin^2\theta}{\sin\theta}$$
$$= \frac{\cos^2\theta + \sin^2\theta}{\sin\theta}$$
$$= \frac{1}{\sin\theta}$$

Quick Practice 9.9 (p. 9.19)

$\because$ $\cos^2\theta = 1 - \sin^2\theta$

$$\therefore \quad \cos\theta = \sqrt{1-\sin^2\theta}$$
$$= \sqrt{1-\left(\frac{5}{13}\right)^2}$$
$$= \sqrt{\frac{144}{169}}$$
$$= \frac{12}{13}$$
$$\tan\theta = \frac{\sin\theta}{\cos\theta}$$
$$= \frac{5}{13}\times\frac{13}{12}$$
$$= \frac{5}{12}$$

$$\therefore \quad (3\sin\theta + 2\cos\theta)\tan^2\theta = \left(3\times\frac{5}{13} + 2\times\frac{12}{13}\right)\times\left(\frac{5}{12}\right)^2$$
$$= 3\times\frac{25}{144}$$
$$= \frac{25}{48}$$

Quick Practice 9.10 (p. 9.19)

$$\tan\theta = \frac{\sin\theta}{\cos\theta}$$
$$\frac{1}{2} = \frac{\sin\theta}{\cos\theta}$$
$$\cos\theta = 2\sin\theta$$
$$\cos^2\theta = 4\sin^2\theta$$
$$1-\sin^2\theta = 4\sin^2\theta$$
$$1 = 5\sin^2\theta$$
$$\sin^2\theta = \frac{1}{5}$$
$$\sin\theta = \frac{1}{\sqrt{5}}\left(\text{or } \frac{\sqrt{5}}{5}\right)$$
$$\cos^2\theta = 1-\sin^2\theta$$
$$\cos\theta = \sqrt{1-\frac{1}{5}}$$
$$= \frac{2}{\sqrt{5}}\left(\text{or } \frac{2\sqrt{5}}{5}\right)$$

Quick Practice 9.11 (p. 9.20)

$$\frac{4\cos\theta - 3\sin\theta}{9\sin\theta} = \frac{4\cos\theta}{9\sin\theta} - \frac{3\sin\theta}{9\sin\theta}$$
$$= \frac{4}{9\tan\theta} - \frac{1}{3}$$
$$= \frac{4}{9}\times\frac{5}{2} - \frac{1}{3}$$
$$= \frac{7}{9}$$

Alternative Solution

$$\tan\theta = \frac{\sin\theta}{\cos\theta}$$
$$\frac{2}{5} = \frac{\sin\theta}{\cos\theta}$$
$$2\cos\theta = 5\sin\theta$$
$$\therefore \quad \frac{4\cos\theta - 3\sin\theta}{9\sin\theta} = \frac{2(5\sin\theta) - 3\sin\theta}{9\sin\theta}$$
$$= \frac{7\sin\theta}{\sin\theta}$$
$$= \underline{\underline{\frac{7}{9}}}$$

Quick Practice 9.12 (p. 9.24)

(a) $\sin\theta = \cos 38°$
$= \cos(90° - 52°)$
$= \sin 52°$
$\therefore \quad \theta = \underline{\underline{52°}}$

(b) $\tan 2\theta = \dfrac{1}{\tan\theta}$
$= \tan(90° - \theta)$
$\therefore \quad 2\theta = 90° - \theta$
$3\theta = 90°$
$\theta = \underline{\underline{30°}}$

Quick Practice 9.13 (p. 9.25)

(a) $\dfrac{\sin(90° - \theta)}{\tan(90° - \theta)} = \dfrac{\cos\theta}{\dfrac{1}{\tan\theta}}$
$= \cos\theta \times \tan\theta$
$= \cos\theta \times \dfrac{\sin\theta}{\cos\theta}$
$= \underline{\underline{\sin\theta}}$

(b) $\tan(90° - \theta)\sin\theta - \sin(90° - \theta) = \dfrac{1}{\tan\theta} \times \sin\theta - \cos\theta$
$= \dfrac{\cos\theta}{\sin\theta} \times \sin\theta - \cos\theta$
$= \underline{\underline{0}}$

Quick Practice 9.14 (p. 9.25)

(a) $\dfrac{\tan 41° \sin 49°}{\cos 49°} = \tan(90° - 49°) \times \dfrac{\sin 49°}{\cos 49°}$
$= \dfrac{1}{\tan 49°} \times \tan 49°$
$= \underline{\underline{1}}$

(b) $(\tan^2 64° + 1)\sin^2 26° = [\tan^2(90° - 26°) + 1]\sin^2 26°$
$= \left(\dfrac{1}{\tan^2 26°} + 1\right)\sin^2 26°$
$= \left(\dfrac{\cos^2 26°}{\sin^2 26°} + 1\right)\sin^2 26°$
$= \cos^2 26° + \sin^2 26°$
$= \underline{\underline{1}}$

Quick Practice 9.15 (p. 9.29)

(a) L.H.S. $= 1 - 2\sin^2\theta$
$= 1 - 2(1 - \cos^2\theta)$
$= 1 - 2 + 2\cos^2\theta$
$= 2\cos^2\theta - 1$
R.H.S. $= 2\cos^2\theta - 1$
$\because$ L.H.S. = R.H.S.
$\therefore \quad 1 - 2\sin^2\theta \equiv 2\cos^2\theta$

(b) L.H.S. $= \dfrac{2\cos\theta}{\tan(90° - \theta)} - \sin\theta$
$= 2\cos\theta\tan\theta - \sin\theta$
$= 2\cos\theta \times \dfrac{\sin\theta}{\cos\theta} - \sin\theta$
$= 2\sin\theta - \sin\theta$
$= \sin\theta$
R.H.S. $= \cos(90° - \theta)$
$= \sin\theta$
$\because$ L.H.S. = R.H.S.
$\therefore \quad \dfrac{2\cos\theta}{\tan(90° - \theta)} - \sin\theta \equiv \cos(90° - \theta)$

Quick Practice 9.16 (p. 9.29)

L.H.S. $= (\sin\theta + \cos\theta)^2 + (\sin\theta - \cos\theta)^2$
$= (\sin^2\theta + 2\sin\theta\cos\theta + \cos^2\theta)$
$+ (\sin^2\theta - 2\sin\theta\cos\theta + \cos^2\theta)$
$= 2(\sin^2\theta + \cos^2\theta)$
$= 2(1)$
$= 2$
R.H.S. $= 2$
$\because$ L.H.S. = R.H.S.
$\therefore \quad (\sin\theta + \cos\theta)^2 + (\sin\theta - \cos\theta)^2 \equiv 2$

Further Practice

Further Practice (p. 9.8)

1. (a) $\tan 60° \cos 30° = \sqrt{3} \times \dfrac{\sqrt{3}}{2}$
$= \underline{\underline{\dfrac{3}{2}}}$

(b) $\dfrac{\tan 45° \cos 45°}{\sin 60°} = \dfrac{1 \times \dfrac{1}{\sqrt{2}}}{\dfrac{\sqrt{3}}{2}}$
$= \dfrac{1}{\sqrt{2}} \times \dfrac{2}{\sqrt{3}}$
$= \underline{\underline{\dfrac{\sqrt{2}}{\sqrt{3}}}} \left(\text{or } \dfrac{\sqrt{6}}{3}\right)$

(c) $\sin^2 45° - \tan^2 30° = \left(\frac{1}{\sqrt{2}}\right)^2 - \left(\frac{1}{\sqrt{3}}\right)^2$

$= \frac{1}{2} - \frac{1}{3}$

$= \underline{\underline{\frac{1}{6}}}$

2. **(a)** $3\tan\theta = \sqrt{3}$

$\tan\theta = \frac{\sqrt{3}}{3}$

$\theta = \underline{\underline{30°}}$

(b) $\sqrt{2}\sin\theta - \tan 45° = 0$

$\sqrt{2}\sin\theta = 1$

$\sin\theta = \frac{1}{\sqrt{2}}$

$\theta = \underline{\underline{45°}}$

(c) $\cos 2\theta - \sin 60° = 0$

$\cos 2\theta = \frac{\sqrt{3}}{2}$

$2\theta = 30°$

$\theta = \underline{\underline{15°}}$

3. **(a)** Consider $\triangle ABC$.

$\tan\angle ABC = \frac{AC}{BC}$

$AC = BC\tan(30° + 30°)$

$= \sqrt{3}\tan 60°$ m

$= \sqrt{3}\times\sqrt{3}$ m

$= \underline{\underline{3\text{m}}}$

(b) Consider $\triangle BCD$.

$\tan 30° = \frac{DC}{BC}$

$DC = BC\tan 30°$

$DC = \sqrt{3}\times\frac{1}{\sqrt{3}}$ m

$= 1$ m

$AD = AC - DC$

$= (3-1)$ m

$= \underline{\underline{2\text{m}}}$

Further Practice (p. 9.21)

1. **(a)** $\cos^2\theta + 1 - \sin^2\theta = \cos^2\theta + \cos^2\theta$

$= \underline{\underline{2\cos^2\theta}}$

(b) $\frac{1}{\tan\theta\cos\theta} + \frac{1}{\sin\theta} = \frac{1}{\frac{\sin\theta}{\cos\theta}\times\cos\theta} + \frac{1}{\sin\theta}$

$= \frac{1}{\sin\theta} + \frac{1}{\sin\theta}$

$= \underline{\underline{\frac{2}{\sin\theta}}}$

(c) $\frac{1-\cos^2\theta}{\tan^2\theta(1-\sin^2\theta)} = \frac{\sin^2\theta}{\left(\frac{\sin\theta}{\cos\theta}\right)^2\times\cos^2\theta}$

$= \frac{\sin^2\theta}{\sin^2\theta}$

$= \underline{\underline{1}}$

2. $\cos\theta = \frac{3}{5}$

$\cos^2\theta = \left(\frac{3}{5}\right)^2$

$1 - \sin^2\theta = \frac{9}{25}$

$\sin^2\theta = 1 - \frac{9}{25}$

$\sin^2\theta = \frac{16}{25}$

$\sin\theta = \underline{\underline{\frac{4}{5}}}$

$\tan\theta = \frac{\sin\theta}{\cos\theta}$

$= \frac{\frac{4}{5}}{\frac{3}{5}}$

$= \underline{\underline{\frac{4}{3}}}$

3. $\because$ $\tan\theta = \frac{\sqrt{3}}{2}$

$\therefore$ $\frac{\sin\theta}{\cos\theta} = \frac{\sqrt{3}}{2}$

$2\sin\theta = \sqrt{3}\cos\theta$

$4\sin^2\theta = 3\cos^2\theta$

$4\sin^2\theta = 3(1-\sin^2\theta)$

$4\sin^2\theta = 3 - 3\sin^2\theta$

$7\sin^2\theta = 3$

$\sin^2\theta = \frac{3}{7}$

$\sin\theta = \underline{\underline{\frac{\sqrt{3}}{\sqrt{7}}}}\left(\text{or } \frac{\sqrt{21}}{7}\right)$

and $\cos\theta = \frac{2}{\sqrt{3}}\sin\theta$

$= \frac{2}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{7}}$

$= \underline{\underline{\frac{2}{\sqrt{7}}}} \left(\text{or } \frac{2\sqrt{7}}{7}\right)$

4. $\frac{\sin\theta}{\sin\theta + \cos\theta} = \frac{\frac{\sin\theta}{\cos\theta}}{\frac{\sin\theta + \cos\theta}{\cos\theta}}$

$= \frac{\frac{\sin\theta}{\cos\theta}}{\frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\cos\theta}}$

$= \frac{\tan\theta}{\tan\theta + 1}$

$= \frac{4}{4+1}$

$= \underline{\underline{\frac{4}{5}}}$

Further Practice (p. 9.26)

1. **(a)** $\cos\theta = \sin 60°$

$= \cos(90° - 60°)$

$= \cos 30°$

$\theta = \underline{\underline{30°}}$

(b) $\sin\theta = \cos 45°$

$= \sin(90° - 45°)$

$= \sin 45°$

$\theta = \underline{\underline{45°}}$

(c) $\tan\theta = \frac{1}{\tan 30°}$

$= \tan(90° - 30°)$

$= \tan 60°$

$\theta = \underline{\underline{60°}}$

2. **(a)** $\frac{1}{\cos^2\theta} - \frac{1}{\tan^2(90° - \theta)} = \frac{1}{\cos^2\theta} - \frac{1}{\frac{1}{\tan^2\theta}}$

$= \frac{1}{\cos^2\theta} - \tan^2\theta$

$= \frac{1}{\cos^2\theta} - \frac{\sin^2\theta}{\cos^2\theta}$

$= \frac{1 - \sin^2\theta}{\cos^2\theta}$

$= \frac{\cos^2\theta}{\cos^2\theta}$

$= \underline{\underline{1}}$

(b) $\sin^2(90° - \theta) - \frac{\sin^2\theta}{\tan^2\theta} = \cos^2\theta - \frac{\sin^2\theta}{\frac{\sin^2\theta}{\cos^2\theta}}$

$= \cos^2\theta - \cos^2\theta$

$= \underline{\underline{0}}$

3. **(a)** $3\cos^2 17° + 3\cos^2 73° + 3$

$= 3\cos^2(90° - 73°) + 3\cos^2 73° + 3$

$= 3\sin^2 73° + 3\cos^2 73° + 3$

$= 3(\sin^2 73° + \cos^2 73°) + 3$

$= 3(1) + 3$

$= \underline{\underline{6}}$

(b) $\frac{\tan 21° \tan 69°}{1 + \tan 21° \tan 69°} = \frac{\tan(90° - 69°)\tan 69°}{1 + \tan(90° - 69°)\tan 69°}$

$= \frac{\frac{1}{\tan 69°} \times \tan 69°}{1 + \frac{1}{\tan 69°} \times \tan 69°}$

$= \frac{1}{1+1}$

$= \underline{\underline{\frac{1}{2}}}$

Exercise

Exercise 9A (p. 9.8)
Level 1

1. $2 - \sin 30° = 2 - \frac{1}{2}$

$= \underline{\underline{\frac{3}{2}}}$

2. $2\tan 45° = 2 \times 1$

$= \underline{\underline{2}}$

3. $\frac{2}{\sin 45°} = \frac{2}{\frac{1}{\sqrt{2}}}$

$= \underline{\underline{2\sqrt{2}}}$

4. $\cos 45° + \sin 45° = \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}$

$= \frac{2}{\sqrt{2}}$

$= \underline{\underline{\sqrt{2}}}$

5. $2\sin 60° - \tan 60° = 2 \times \frac{\sqrt{3}}{2} - \sqrt{3}$

$= \sqrt{3} - \sqrt{3}$

$= \underline{\underline{0}}$

6. $\tan 30° \sin 60° = \frac{1}{\sqrt{3}} \times \frac{\sqrt{3}}{2}$

$= \underline{\underline{\frac{1}{2}}}$

7. $\frac{\sin 30°}{\tan 45°} = \frac{\frac{1}{2}}{1}$

$= \underline{\underline{\frac{1}{2}}}$

8. $\sin^2 60° = \left(\frac{\sqrt{3}}{2}\right)^2$

$= \underline{\underline{\frac{3}{4}}}$

9. $\frac{1}{\sin 30° + \cos^2 30°} = \frac{1}{\frac{1}{2} + \left(\frac{\sqrt{3}}{2}\right)^2}$

$= \frac{1}{\frac{1}{2} + \frac{3}{4}}$

$= \frac{1}{\frac{5}{4}}$

$= \underline{\underline{\frac{4}{5}}}$

10. $\frac{\sin 45° - \cos 45°}{\tan 45°} = \frac{\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}}}{1}$

$= \underline{\underline{0}}$

11. $\frac{\tan 60° \cos 45°}{\sin 45°} = \frac{\sqrt{3} \times \frac{1}{\sqrt{2}}}{\frac{1}{\sqrt{2}}}$

$= \underline{\underline{\sqrt{3}}}$

12. $2\cos 60° - \sin 60° \tan 60° = 2 \times \frac{1}{2} - \frac{\sqrt{3}}{2} \times \sqrt{3}$

$= 1 - \frac{3}{2}$

$= \underline{\underline{-\frac{1}{2}}}$

13. $2\sin\theta = \sqrt{2}$

$\sin\theta = \frac{\sqrt{2}}{2}$

$\theta = \underline{\underline{45°}}$

14. $\tan\theta - \sqrt{3} = 0$

$\tan\theta = \sqrt{3}$

$\theta = \underline{\underline{60°}}$

15. $2 - \cos\theta = \frac{3}{2}$

$\cos\theta = \frac{1}{2}$

$\theta = \underline{\underline{60°}}$

16. $\tan\theta = \frac{1}{2\sin 60°}$

$\tan\theta = \frac{1}{2 \times \frac{\sqrt{3}}{2}}$

$\tan\theta = \frac{1}{\sqrt{3}}$

$\theta = \underline{\underline{30°}}$

17. $2\sin 30° - \tan\theta = 0$

$2 \times \frac{1}{2} - \tan\theta = 0$

$\tan\theta = 1$

$\theta = \underline{\underline{45°}}$

18. $\sqrt{2}\cos\theta - \cos 45° = 0$

$\sqrt{2}\cos\theta = \frac{1}{\sqrt{2}}$

$\cos\theta = \frac{1}{\sqrt{2}} \times \frac{1}{\sqrt{2}}$

$\cos\theta = \frac{1}{2}$

$\theta = \underline{\underline{60°}}$

19. **(a)** $\cos\theta = \frac{AC}{AB}$

$= \frac{4}{8}$

$= \frac{1}{2}$

$\theta = \underline{\underline{60°}}$

(b) $\sin 30° = \frac{AB}{AC}$

$= \frac{x}{10}$

$x = 10\sin 30°$

$= 10 \times \frac{1}{2}$

$= \underline{\underline{5}}$

(c) $\tan 45° = \dfrac{BC}{AC}$

$= \dfrac{3}{x}$

$x = \dfrac{3}{\tan 45°}$

$= \dfrac{3}{1}$

$= \underline{\underline{3}}$

Level 2

20. $\dfrac{\tan 30°}{\sin 60°} + \cos^2 45° = \dfrac{\frac{1}{\sqrt{3}}}{\frac{\sqrt{3}}{2}} + \left(\dfrac{1}{\sqrt{2}}\right)^2$

$= \dfrac{2}{3} + \dfrac{1}{2}$

$= \underline{\underline{\dfrac{7}{6}}}$

21. $\cos^2 30° - \sin^2 45° + \cos 60° = \left(\dfrac{\sqrt{3}}{2}\right)^2 - \left(\dfrac{1}{\sqrt{2}}\right)^2 + \dfrac{1}{2}$

$= \dfrac{3}{4} - \dfrac{1}{2} + \dfrac{1}{2}$

$= \underline{\underline{\dfrac{3}{4}}}$

22. $\sin 60° \cos 45° - \cos 60° \sin 45° = \dfrac{\sqrt{3}}{2} \times \dfrac{1}{\sqrt{2}} - \dfrac{1}{2} \times \dfrac{1}{\sqrt{2}}$

$= \dfrac{\sqrt{3} - 1}{2\sqrt{2}}$

$= \underline{\underline{\dfrac{\sqrt{6} - \sqrt{2}}{4}}}$

23. $(\sin 30° + \sin 45°)\left(\tan 60° + \dfrac{1}{\tan 30°}\right)$

$= \left(\dfrac{1}{2} + \dfrac{\sqrt{2}}{2}\right)\left[\sqrt{3} + \dfrac{1}{\left(\frac{1}{\sqrt{3}}\right)}\right]$

$= \left(\dfrac{1}{2} + \dfrac{\sqrt{2}}{2}\right)(\sqrt{3} + \sqrt{3})$

$= \left(\dfrac{1 + \sqrt{2}}{2}\right) \times 2\sqrt{3}$

$= \underline{\underline{\sqrt{3} + \sqrt{6}}}$

24. $\sqrt{3} - 2\cos 3\theta = 0$

$\cos 3\theta = \dfrac{\sqrt{3}}{2}$

$3\theta = 30°$

$\theta = \underline{\underline{10°}}$

25. $\sqrt{3} \tan \dfrac{\theta}{2} - 1 = 0$

$\tan \dfrac{\theta}{2} = \dfrac{1}{\sqrt{3}}$

$\dfrac{\theta}{2} = 30°$

$\theta = \underline{\underline{60°}}$

26. $\sin\theta \sin 45° - \cos 60° = 0$

$\dfrac{1}{\sqrt{2}} \sin\theta = \dfrac{1}{2}$

$\sin\theta = \dfrac{\sqrt{2}}{2}$

$\theta = \underline{\underline{45°}}$

27. $\tan 2\theta \tan 60° - \tan 45° = 0$

$\sqrt{3} \tan 2\theta - 1 = 0$

$\tan 2\theta = \dfrac{1}{\sqrt{3}}$

$2\theta = 30°$

$\theta = \underline{\underline{15°}}$

28. Consider $\triangle ABC$.

$\cos 60° = \dfrac{AC}{BC}$

$= \dfrac{1}{x}$

$x = \dfrac{1}{\frac{1}{2}}$

$= \underline{\underline{2}}$

Consider $\triangle CBD$.

$\cos 45° = \dfrac{BD}{BC}$

$= \dfrac{y}{2}$

$y = 2 \times \dfrac{\sqrt{2}}{2}$

$= \underline{\underline{\sqrt{2}}}$

29. Consider $\triangle CDA$.

$\because \quad \angle CAD = 45° - 15°$

$= 30°$

$\therefore \quad \tan \angle CAD = \dfrac{CD}{AD}$

$\tan 30° = \dfrac{x}{6}$

$x = 6 \times \dfrac{\sqrt{3}}{3}$

$= \underline{\underline{2\sqrt{3}}}$

Consider $\triangle BDA$.

$$\tan 45° = \frac{BD}{6}$$
$$BD = 6\tan 45°$$
$$y + 2\sqrt{3} = 6(1)$$
$$y = \underline{\underline{6 - 2\sqrt{3}}}$$

30. Consider $\triangle ACD$.

$$\sin 30° = \frac{AC}{10}$$
$$AC = 10\sin 30°$$
$$= 10 \times \frac{1}{2}$$
$$= 5$$

Consider $\triangle ABC$.

$$\tan\theta = \frac{AC}{5}$$
$$= \frac{5}{5}$$
$$= 1$$
$$\theta = \underline{\underline{45°}}$$
$$\cos 45° = \frac{5}{x}$$
$$x = \frac{5}{\frac{1}{\sqrt{2}}}$$
$$= \underline{\underline{5\sqrt{2}}}$$

31. Consider $\triangle ACD$.

$$\sin\theta = \frac{2\sqrt{3}}{4}$$
$$= \frac{\sqrt{3}}{2}$$
$$\theta = \underline{\underline{60°}}$$
$$\cos 60° = \frac{CD}{4}$$
$$CD = 4\cos 60°$$
$$= 4 \times \frac{1}{2}$$
$$= 2$$

Consider $\triangle ABD$.

$$\tan 45° = \frac{2\sqrt{3}}{x + CD}$$
$$x + 2 = 2\sqrt{3}$$
$$x = \underline{\underline{2\sqrt{3} - 2}}$$

32. **(a)** $\angle CAD = \angle ECA = 45°$ (alt. $\angle$s, EF // AB)

Consider $\triangle CAD$.

$$\sin\angle CAD = \frac{CD}{AC}$$
$$\sin 45° = \frac{CD}{6\text{ m}}$$
$$CD = 6 \times \frac{\sqrt{2}}{2}\text{ m}$$
$$= \underline{\underline{3\sqrt{2}\text{ m}}}$$

(b) $\angle DBC = \angle BCF = 30°$ (alt. $\angle$s, EF // AB)

Consider $\triangle CDB$.

$$\sin\angle DBC = \frac{CD}{CB}$$
$$\sin 30° = \frac{3\sqrt{2}\text{ m}}{CB}$$
$$CB = \frac{3\sqrt{2}}{\frac{1}{2}}\text{ m}$$
$$= \underline{\underline{6\sqrt{2}\text{ m}}}$$

33. **(a)** Consider $\triangle ABC$.

$$\sin\theta = \frac{2\sqrt{3}}{4}$$
$$= \frac{\sqrt{3}}{2}$$
$$\theta = \underline{\underline{60°}}$$

(b) Consider $\triangle CED$.

$\angle DCE = 60°$ (vert. opp. $\angle$s)

$$\cos 60° = \frac{3\text{ cm}}{CD}$$
$$CD = \frac{3}{\cos 60°}\text{ cm}$$
$$= \frac{3}{\frac{1}{2}}\text{ cm}$$
$$= \underline{\underline{6\text{ cm}}}$$
$$\tan 60° = \frac{DE}{3\text{ cm}}$$
$$DE = 3\tan 60°\text{ cm}$$
$$= \underline{\underline{3\sqrt{3}\text{ cm}}}$$

34. **(a)** $\angle ACF = \angle CBD = 30°$ (corr. $\angle$s, CF // BE)

Consider $\triangle ACF$.

$$\tan 30° = \frac{AF}{9\text{ cm}}$$
$$AF = 9 \times \frac{\sqrt{3}}{3}\text{ cm}$$
$$= 3\sqrt{3}\text{ cm}$$
$$AE = AF + FE$$
$$= AF + CD$$
$$= \underline{\underline{(3\sqrt{3} + 6)\text{ cm}}}$$

(b) Consider $\triangle ABE$.

$$\sin 30° = \frac{AE}{AB}$$
$$= \frac{(3\sqrt{3} + 6)\text{ cm}}{AB}$$
$$AB = \frac{3\sqrt{3} + 6}{\frac{1}{2}}\text{ cm}$$
$$= \underline{\underline{(6\sqrt{3} + 12)\text{ cm}}}$$

35. Consider $\triangle ABE$.

$$\sin 30° = \frac{AE}{38\text{ m}}$$
$$AE = 38 \times \frac{1}{2}\text{ m}$$
$$= 19\text{ m}$$

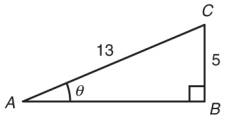
Let h m be the height of the box will be lifted.

$$\sin 60° = \frac{AE + h}{AB}$$
$$AB \sin 60° = AE + h$$
$$38 \times \frac{\sqrt{3}}{2} = 19 + h$$
$$h = 19\sqrt{3} - 19$$

$\therefore$ The box will be lifted $(19\sqrt{3} - 19)$ m.

Exercise 9B (p. 9.14)
Level 1

1. Construct $\triangle ABC$ as shown with $\sin\theta = \frac{5}{13}$.



By Pythagoras' theorem,

$$AB = \sqrt{AC^2 - BC^2}$$
$$= \sqrt{13^2 - 5^2}$$
$$= 12$$

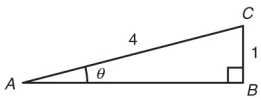
By definition,

$$\cos\theta = \frac{AB}{AC} = \underline{\underline{\frac{12}{13}}}$$
$$\tan\theta = \frac{BC}{AB} = \underline{\underline{\frac{5}{12}}}$$

2. $\sin\theta = 0.25$

$$= \frac{25}{100}$$
$$= \frac{1}{4}$$

Construct $\triangle ABC$ as shown with $\sin\theta = \frac{1}{4}$.



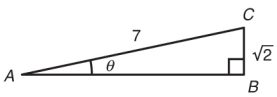
By Pythagoras' theorem,

$$AB = \sqrt{AC^2 - BC^2}$$
$$= \sqrt{4^2 - 1^2}$$
$$= \sqrt{15}$$

By definition,

$$\cos\theta = \frac{AB}{AC} = \underline{\underline{\frac{\sqrt{15}}{4}}}$$
$$\tan\theta = \frac{BC}{AB} = \underline{\underline{\frac{1}{\sqrt{15}}}}\left(\text{or } \frac{\sqrt{15}}{15}\right)$$

3. Construct $\triangle ABC$ as shown with $\sin\theta = \frac{\sqrt{2}}{7}$.



By Pythagoras' theorem,

$$AB = \sqrt{AC^2 - BC^2}$$
$$= \sqrt{7^2 - (\sqrt{2})^2}$$
$$= \sqrt{47}$$

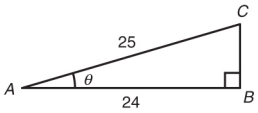
By definition,

$$\cos\theta = \frac{AB}{AC} = \underline{\underline{\frac{\sqrt{47}}{7}}}$$
$$\tan\theta = \frac{BC}{AB} = \underline{\underline{\sqrt{\frac{2}{47}}}}\left(\text{or } \frac{\sqrt{94}}{47}\right)$$

4. $\cos\theta = 0.96$

$$= \frac{96}{100}$$
$$= \frac{24}{25}$$

Construct $\triangle ABC$ as shown with $\cos\theta = \frac{24}{25}$.



By Pythagoras' theorem,

$$BC = \sqrt{AC^2 - AB^2}$$
$$= \sqrt{25^2 - 24^2}$$
$$= 7$$

By definition,

$$\sin\theta = \frac{BC}{AC} = \underline{\underline{\frac{7}{25}}}$$
$$\tan\theta = \frac{BC}{AB} = \underline{\underline{\frac{7}{24}}}$$

5. Construct $\triangle ABC$ as shown with $\cos\theta = \frac{1}{7}$.

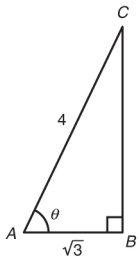
By Pythagoras' theorem,

$$BC = \sqrt{AC^2 - AB^2}$$
$$= \sqrt{7^2 - 1^2}$$
$$= \sqrt{48}(\text{or } 4\sqrt{3})$$

By definition,

$$\sin\theta = \frac{BC}{AC} = \underline{\underline{\frac{\sqrt{48}}{7}}}\left(\text{or } \frac{4\sqrt{3}}{7}\right)$$
$$\tan\theta = \frac{BC}{AB} = \frac{\sqrt{48}}{1} = \underline{\underline{\sqrt{48}}}(\text{or } 4\sqrt{3})$$

6. Construct $\triangle ABC$ as shown with $\cos\theta = \frac{\sqrt{3}}{4}$.



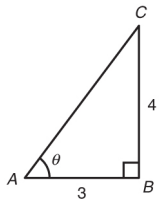
By Pythagoras' theorem,

$$BC = \sqrt{AC^2 - AB^2}$$
$$= \sqrt{4^2 - (\sqrt{3})^2}$$
$$= \sqrt{13}$$

By definition,

$$\sin\theta = \frac{BC}{AC} = \underline{\underline{\frac{\sqrt{13}}{4}}}$$
$$\tan\theta = \frac{BC}{AB} = \underline{\underline{\sqrt{\frac{13}{3}}}}\left(\text{or } \frac{\sqrt{39}}{3}\right)$$

7. Construct $\triangle ABC$ as shown with $\tan\theta = \frac{4}{3}$.



By Pythagoras' theorem,

$$AC = \sqrt{AB^2 + BC^2}$$
$$= \sqrt{3^2 + 4^2}$$
$$= 5$$

By definition,

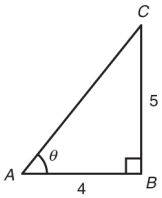
$$\sin\theta = \frac{BC}{AC} = \underline{\underline{\frac{4}{5}}}$$

$$\cos\theta = \frac{AB}{AC} = \underline{\underline{\frac{3}{5}}}$$

8. $\tan\theta = 1.25$

$$= \frac{125}{100}$$
$$= \frac{5}{4}$$

Construct $\triangle ABC$ as shown with $\tan\theta = \frac{5}{4}$.



By Pythagoras' theorem,

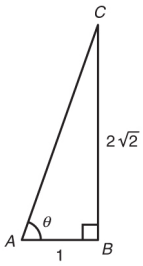
$$AC = \sqrt{AB^2 + BC^2}$$
$$= \sqrt{4^2 + 5^2}$$
$$= \sqrt{41}$$

By definition,

$$\sin\theta = \frac{BC}{AC} = \underline{\underline{\frac{5}{\sqrt{41}}}} \left(\text{or } \frac{5\sqrt{41}}{41}\right)$$

$$\cos\theta = \frac{AB}{AC} = \underline{\underline{\frac{4}{\sqrt{41}}}} \left(\text{or } \frac{4\sqrt{41}}{41}\right)$$

9. Construct $\triangle ABC$ as shown with $\tan\theta = \frac{2\sqrt{2}}{1}$.



By Pythagoras' theorem,

$$AC = \sqrt{AB^2 + BC^2}$$
$$= \sqrt{1^2 + (2\sqrt{2})^2}$$
$$= 3$$

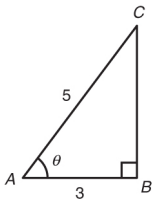
By definition,

$$\sin\theta = \frac{BC}{AC} = \underline{\underline{\frac{2\sqrt{2}}{3}}}$$

$$\cos\theta = \frac{AB}{AC} = \underline{\underline{\frac{1}{3}}}$$

Level 2

10. Construct $\triangle ABC$ as shown with $\cos\theta = \frac{3}{5}$.



By Pythagoras' theorem,

$$BC = \sqrt{AC^2 - AB^2}$$
$$= \sqrt{5^2 - 3^2}$$
$$= 4$$

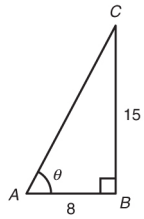
By definition,

$$\sin\theta = \frac{BC}{AC} = \frac{4}{5}$$

$$\tan\theta = \frac{BC}{AB} = \frac{4}{3}$$

$$\therefore \quad \sin\theta\tan\theta = \frac{4}{5} \times \frac{4}{3}$$
$$= \underline{\underline{\frac{16}{15}}}$$

11. Construct $\triangle ABC$ as shown with $\tan\theta = \frac{15}{8}$.



By Pythagoras' theorem,

$$AC = \sqrt{AB^2 + BC^2}$$
$$= \sqrt{8^2 + 15^2}$$
$$= 17$$

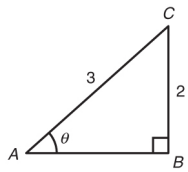
By definition,

$$\sin\theta = \frac{BC}{AC} = \frac{15}{17}$$

$$\cos\theta = \frac{AB}{AC} = \frac{8}{17}$$

$$\therefore \quad 3\sin\theta + 5\cos\theta = 3 \times \frac{15}{17} + 5 \times \frac{8}{17}$$
$$= \frac{85}{17}$$
$$= \underline{\underline{5}}$$

12. Construct $\triangle ABC$ as shown with $\sin\theta = \frac{2}{3}$.



By Pythagoras' theorem,

$$AB = \sqrt{AC^2 - BC^2}$$
$$= \sqrt{3^2 - 2^2}$$
$$= \sqrt{5}$$

By definition,

$$\tan\theta = \frac{BC}{AB} = \frac{2}{\sqrt{5}}$$

$$\therefore \quad \tan^2\theta = \left(\frac{2}{\sqrt{5}}\right)^2$$
$$= \underline{\underline{\frac{4}{5}}}$$

13. Construct $\triangle ABC$ as shown with $\tan\theta = \frac{5}{2}$.

By Pythagoras' theorem,

$$AC = \sqrt{AB^2 + BC^2}$$
$$= \sqrt{2^2 + 5^2}$$
$$= \sqrt{29}$$

By definition,

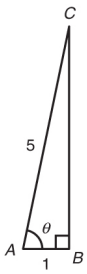
$$\sin\theta = \frac{BC}{AC} = \frac{5}{\sqrt{29}}$$

$$\cos\theta = \frac{AB}{AC} = \frac{2}{\sqrt{29}}$$

$$\therefore \quad \frac{1}{\sin\theta - \cos\theta} = \frac{1}{\frac{5}{\sqrt{29}} - \frac{2}{\sqrt{29}}}$$
$$= \frac{1}{\frac{3}{\sqrt{29}}}$$
$$= \underline{\underline{\frac{\sqrt{29}}{3}}}$$

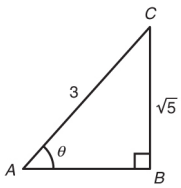
14. $\cos\theta = 0.2$
$$= \frac{2}{10}$$
$$= \frac{1}{5}$$

Construct $\triangle ABC$ as shown with $\cos\theta = \frac{1}{5}$.



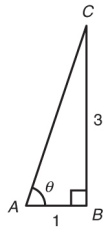
By Pythagoras' theorem,
$$BC = \sqrt{AC^2 - AB^2}$$
$$= \sqrt{5^2 - 1^2}$$
$$= \sqrt{24}$$
By definition,
$$\sin\theta = \frac{BC}{AC} = \frac{\sqrt{24}}{5}$$
$$\tan\theta = \frac{BC}{AB} = \frac{\sqrt{24}}{1} = \sqrt{24}$$
$$\therefore \quad \frac{\tan\theta}{\sin\theta} = \frac{\sqrt{24}}{\frac{\sqrt{24}}{5}}$$
$$= \underline{\underline{5}}$$

15. Construct $\triangle ABC$ as shown with $\sin\theta = \frac{\sqrt{5}}{3}$.



By Pythagoras' theorem,
$$AB = \sqrt{AC^2 - BC^2}$$
$$= \sqrt{3^2 - (\sqrt{5})^2}$$
$$= 2$$
By definition,
$$\tan\theta = \frac{BC}{AB} = \frac{\sqrt{5}}{2}$$
$$\therefore \quad \frac{\tan\theta}{\sqrt{1+\tan^2\theta}} = \frac{\frac{\sqrt{5}}{2}}{\sqrt{1+\left(\frac{\sqrt{5}}{2}\right)^2}}$$
$$= \frac{\frac{\sqrt{5}}{2}}{\sqrt{\frac{9}{4}}}$$
$$= \frac{\sqrt{5}}{2} \times \frac{2}{3}$$
$$= \underline{\underline{\frac{\sqrt{5}}{3}}}$$

16. Construct $\triangle ABC$ as shown with $\tan\theta = \frac{3}{1}$.



By Pythagoras' theorem,
$$AC = \sqrt{AB^2 + BC^2}$$
$$= \sqrt{1^2 + 3^2}$$
$$= \sqrt{10}$$
By definition,
$$\sin\theta = \frac{BC}{AC} = \frac{3}{\sqrt{10}}$$
$$\cos\theta = \frac{AB}{AC} = \frac{1}{\sqrt{10}}$$
$$\therefore \quad \frac{\cos\theta - 2\sin\theta}{\sin\theta + \cos\theta} = \frac{\frac{1}{\sqrt{10}} - 2\times\frac{3}{\sqrt{10}}}{\frac{3}{\sqrt{10}} + \frac{1}{\sqrt{10}}}$$
$$= \frac{\frac{-5}{\sqrt{10}}}{\frac{4}{\sqrt{10}}}$$
$$= \underline{\underline{-\frac{5}{4}}}$$

Exercise 9C (p. 9.21)
Level 1

1. $$\frac{\cos\theta\tan\theta}{\sin\theta} = \frac{\cos\theta \times \frac{\sin\theta}{\cos\theta}}{\sin\theta}$$
$$= \frac{\sin\theta}{\sin\theta}$$
$$= \underline{\underline{1}}$$

2. $$\frac{\tan\theta}{\sin\theta\cos\theta} = \frac{\frac{\sin\theta}{\cos\theta}}{\sin\theta\cos\theta}$$
$$= \frac{\sin\theta}{\cos\theta} \times \frac{1}{\sin\theta\cos\theta}$$
$$= \underline{\underline{\frac{1}{\cos^2\theta}}}$$

3. $$5 - 5\cos^2\theta = 5(1 - \cos^2\theta)$$
$$= \underline{\underline{5\sin^2\theta}}$$

4. $$1 - \frac{\sin\theta\cos\theta}{\tan\theta} = 1 - \frac{\sin\theta\cos\theta}{\frac{\sin\theta}{\cos\theta}}$$
$$= 1 - \sin\theta\cos\theta \times \frac{\cos\theta}{\sin\theta}$$
$$= 1 - \cos^2\theta$$
$$= \underline{\underline{\sin^2\theta}}$$

5. $\dfrac{\sin^2\theta}{\cos\theta}-\dfrac{1}{\cos\theta}=\dfrac{\sin^2\theta-1}{\cos\theta}$

$=\dfrac{-\cos^2\theta}{\cos\theta}$

$=\underline{\underline{-\cos\theta}}$

6. $\dfrac{1}{\tan\theta}-\dfrac{\cos^2\theta}{\tan\theta}=\dfrac{1-\cos^2\theta}{\tan\theta}$

$=\dfrac{\sin^2\theta}{\dfrac{\sin\theta}{\cos\theta}}$

$=\sin^2\theta\times\dfrac{\cos\theta}{\sin\theta}$

$=\underline{\underline{\sin\theta\cos\theta}}$

7. $\cos^3\theta+\cos\theta\sin^2\theta=\cos\theta(\cos^2\theta+\sin^2\theta)$

$=\cos\theta\times1$

$=\underline{\underline{\cos\theta}}$

8. $\dfrac{1}{\cos\theta}-\cos\theta=\dfrac{1-\cos^2\theta}{\cos\theta}$

$=\underline{\underline{\dfrac{\sin^2\theta}{\cos\theta}}}$

9. $1+\dfrac{1}{\tan^2\theta}=1+\dfrac{\cos^2\theta}{\sin^2\theta}$

$=\dfrac{\sin^2\theta+\cos^2\theta}{\sin^2\theta}$

$=\underline{\underline{\dfrac{1}{\sin^2\theta}}}$

10. $\sin\theta\tan\theta+\cos\theta=\sin\theta\dfrac{\sin\theta}{\cos\theta}+\cos\theta$

$=\dfrac{\sin^2\theta}{\cos\theta}+\cos\theta$

$=\dfrac{\sin^2\theta+\cos^2\theta}{\cos\theta}$

$=\underline{\underline{\dfrac{1}{\cos\theta}}}$

11. $(\cos\theta+1)(\cos\theta-1)=\cos^2\theta-1$

$=\underline{\underline{-\sin^2\theta}}$

12. $\dfrac{\sin^2\theta}{1-\sin^2\theta}=\dfrac{\sin^2\theta}{\cos^2\theta}$

$=\underline{\underline{\tan^2\theta}}$

13. $\because\ \cos^2\theta=1-\sin^2\theta$

$\therefore\ \cos\theta=\sqrt{1-\sin^2\theta}$

$=\sqrt{1-\left(\dfrac{1}{3}\right)^2}$

$=\sqrt{1-\dfrac{1}{9}}$

$=\underline{\underline{\dfrac{\sqrt{8}}{3}}}\left(\text{or }\dfrac{2\sqrt{2}}{3}\right)$

14. $\sin^2\theta=1-\cos^2\theta$

$1+\sin^2\theta=2-\cos^2\theta$

$=2-\left(\dfrac{1}{3}\right)^2$

$=2-\dfrac{1}{9}$

$=\underline{\underline{\dfrac{17}{9}}}$

15. $\because\ \sin^2\theta=1-\cos^2\theta$

$\therefore\ \sin\theta=\sqrt{1-\cos^2\theta}$

$=\sqrt{1-\left(\dfrac{15}{17}\right)^2}$

$=\sqrt{\dfrac{64}{289}}$

$=\underline{\underline{\dfrac{8}{17}}}$

and $\tan\theta=\dfrac{\sin\theta}{\cos\theta}$

$=\dfrac{8}{17}\times\dfrac{17}{15}$

$=\underline{\underline{\dfrac{8}{15}}}$

16. $\tan\theta=\dfrac{\sin\theta}{\cos\theta}$

$4=\dfrac{\sin\theta}{\cos\theta}$

$4\cos\theta=\sin\theta$

$16\cos^2\theta=\sin^2\theta$

$16(1-\sin^2\theta)=\sin^2\theta$

$16-16\sin^2\theta=\sin^2\theta$

$17\sin^2\theta=16$

$\sin^2\theta=\dfrac{16}{17}$

$\sin\theta=\underline{\underline{\dfrac{4}{\sqrt{17}}}}\left(\text{or }\dfrac{4\sqrt{17}}{17}\right)$

$\because\ \cos^2\theta=1-\sin^2\theta$

$\therefore\ \cos\theta=\sqrt{1-\dfrac{16}{17}}$

$=\sqrt{\dfrac{1}{17}}$

$=\underline{\underline{\dfrac{1}{\sqrt{17}}}}\left(\text{or }\dfrac{\sqrt{17}}{17}\right)$

17. (a) $\because \quad \cos^2\theta = 1 - \sin^2\theta$

$$\therefore \quad \cos\theta = \sqrt{1-\sin^2\theta}$$
$$= \sqrt{1-\left(\frac{20}{29}\right)^2}$$
$$= \sqrt{\frac{441}{841}}$$
$$= \underline{\underline{\frac{21}{29}}}$$

$$\tan\theta = \frac{\sin\theta}{\cos\theta}$$
$$= \frac{\frac{20}{29}}{\frac{21}{29}}$$
$$= \underline{\underline{\frac{20}{21}}}$$

(b) $3\tan\theta - 2 = 3\times\frac{20}{21} - 2$

$$= \underline{\underline{\frac{6}{7}}}$$

Level 2

18. $1-(\cos\theta+\sin\theta)^2 = 1-(\cos^2\theta + 2\cos\theta\sin\theta + \sin^2\theta)$

$$= 1-(1+2\cos\theta\sin\theta)$$
$$= \underline{\underline{-2\sin\theta\cos\theta}}$$

19. $\tan^2\theta(1+\sin\theta)(1-\sin\theta) = \tan^2\theta(1-\sin^2\theta)$

$$= \frac{\sin^2\theta}{\cos^2\theta}\cos^2\theta$$
$$= \underline{\underline{\sin^2\theta}}$$

20. $\frac{1-(\sin\theta-\cos\theta)^2}{\tan\theta} = \frac{1-(\sin^2\theta - 2\sin\theta\cos\theta + \cos^2\theta)}{\tan\theta}$

$$= \frac{1-(1-2\sin\theta\cos\theta)}{\tan\theta}$$
$$= \frac{2\sin\theta\cos\theta}{\frac{\sin\theta}{\cos\theta}}$$
$$= \underline{\underline{2\cos^2\theta}}$$

21. $\frac{\sqrt{(1-\sin\theta)(1+\sin\theta)}}{\cos^2\theta} = \frac{\sqrt{1-\sin^2\theta}}{\cos^2\theta}$

$$= \frac{\cos\theta}{\cos^2\theta}$$
$$= \underline{\underline{\frac{1}{\cos\theta}}}$$

22. $\frac{1}{\cos^2\theta} - \tan^2\theta - \sin^2\theta = \frac{1}{\cos^2\theta} - \frac{\sin^2\theta}{\cos^2\theta} - \sin^2\theta$

$$= \frac{1-\sin^2\theta}{\cos^2\theta} - \sin^2\theta$$
$$= \frac{\cos^2\theta}{\cos^2\theta} - \sin^2\theta$$
$$= 1-\sin^2\theta$$
$$= \underline{\underline{\cos^2\theta}}$$

23. $\frac{1}{\tan\theta} + \frac{\sin\theta}{1+\cos\theta} = \frac{\cos\theta}{\sin\theta} + \frac{\sin\theta}{1+\cos\theta}$

$$= \frac{\cos\theta + \cos^2\theta + \sin^2\theta}{\sin\theta(1+\cos\theta)}$$
$$= \frac{\cos\theta+1}{\sin\theta(1+\cos\theta)}$$
$$= \underline{\underline{\frac{1}{\sin\theta}}}$$

24. $\frac{2\sin\theta}{\sin\theta+2\cos\theta} = \frac{\frac{2\sin\theta}{\cos\theta}}{\frac{\sin\theta}{\cos\theta}+\frac{2\cos\theta}{\cos\theta}}$

$$= \frac{2\tan\theta}{\tan\theta+2}$$
$$= \frac{2\times\frac{2}{5}}{\frac{2}{5}+2}$$
$$= \frac{\frac{4}{5}}{\frac{12}{5}}$$
$$= \underline{\underline{\frac{1}{3}}}$$

25. $\frac{3\sin\theta+4\cos\theta}{2\sin\theta+6\cos\theta} = \frac{\frac{3\sin\theta}{\cos\theta}+\frac{4\cos\theta}{\cos\theta}}{\frac{2\sin\theta}{\cos\theta}+\frac{6\cos\theta}{\cos\theta}}$

$$= \frac{3\tan\theta+4}{2\tan\theta+6}$$
$$= \frac{3\times\frac{3}{4}+4}{2\times\frac{3}{4}+6}$$
$$= \frac{\frac{25}{4}}{\frac{30}{4}}$$
$$= \underline{\underline{\frac{5}{6}}}$$

26. $\cos\theta + \tan\theta\sin\theta = \cos\theta + \dfrac{\sin\theta}{\cos\theta}\sin\theta$

$= \dfrac{\cos^2\theta + \sin^2\theta}{\cos\theta}$

$= \dfrac{1}{\cos\theta}$

$= \underline{\underline{\dfrac{5}{4}}}$

27. $\dfrac{\tan\theta + \cos\theta}{\tan\theta} + \sin\theta = \dfrac{\dfrac{\sin\theta}{\cos\theta} + \cos\theta}{\dfrac{\sin\theta}{\cos\theta}} + \sin\theta$

$= \dfrac{\sin\theta + \cos^2\theta}{\sin\theta} + \sin\theta$

$= \dfrac{\sin\theta + \cos^2\theta + \sin^2\theta}{\sin\theta}$

$= \dfrac{\sin\theta + 1}{\sin\theta}$

$= 1 + \dfrac{1}{\sin\theta}$

$= 1 + \dfrac{25}{24}$

$= \underline{\underline{\dfrac{49}{24}}}$

Exercise 9D (p. 9.26)
Level 1

1. $\sin\theta = \cos 78°$

$= \sin(90° - 78°)$

$= \sin 12°$

$\therefore \quad \theta = \underline{\underline{12°}}$

2. $\sin\theta = \cos 12°$

$= \sin(90° - 12°)$

$= \sin 78°$

$\therefore \quad \theta = \underline{\underline{78°}}$

3. $\cos\theta = \sin 53.2°$

$= \cos(90° - 53.2°)$

$= \cos 36.8°$

$\therefore \quad \theta = \underline{\underline{36.8°}}$

4. $\cos\theta = \sin 7.3°$

$= \cos(90° - 7.3°)$

$= \cos 82.7°$

$\therefore \quad \theta = \underline{\underline{82.7°}}$

5. $\tan\theta = \dfrac{1}{\tan 70°}$

$= \tan(90° - 70°)$

$= \tan 20°$

$\therefore \quad \theta = \underline{\underline{20°}}$

6. $\tan\theta = \dfrac{\cos 15°}{\sin 15°}$

$= \dfrac{1}{\tan 15°}$

$= \tan(90° - 15°)$

$= \tan 75°$

$\therefore \quad \theta = \underline{\underline{75°}}$

7. $\dfrac{\cos(90° - \theta)}{\sin(90° - \theta)} \times \dfrac{1}{\tan\theta} = \dfrac{\sin\theta}{\cos\theta} \times \dfrac{\cos\theta}{\sin\theta}$

$= \underline{\underline{1}}$

8. $\cos\theta \times \dfrac{2}{\tan(90° - \theta)} = \cos\theta \times 2\tan\theta$

$= \cos\theta \times \dfrac{2\sin\theta}{\cos\theta}$

$= \underline{\underline{2\sin\theta}}$

9. $\dfrac{3\cos\theta}{\tan(90° - \theta)} - 2\cos(90° - \theta) = 3\cos\theta\tan\theta - 2\sin\theta$

$= 3\cos\theta \times \dfrac{\sin\theta}{\cos\theta} - 2\sin\theta$

$= 3\sin\theta - 2\sin\theta$

$= \underline{\underline{\sin\theta}}$

10. $\dfrac{1}{\cos\theta\sin(90° - \theta)} - \dfrac{\tan\theta}{\tan(90° - \theta)}$

$= \dfrac{1}{\cos^2\theta} - \tan^2\theta$

$= \dfrac{1}{\cos^2\theta} - \dfrac{\sin^2\theta}{\cos^2\theta}$

$= \dfrac{1 - \sin^2\theta}{\cos^2\theta}$

$= \dfrac{\cos^2\theta}{\cos^2\theta}$

$= \underline{\underline{1}}$

11. $\dfrac{\sin 36°}{2\cos 54°} = \dfrac{\sin 36°}{2\sin(90° - 54°)}$

$= \dfrac{\sin 36°}{2\sin 36°}$

$= \underline{\underline{\dfrac{1}{2}}}$

12. $\sin 27° - \cos 63° = \sin 27° - \sin(90° - 63°)$

$= \sin 27° - \sin 27°$

$= \underline{\underline{0}}$

13. $1 + \tan 38°\tan 52° = 1 + \tan 38° \dfrac{1}{\tan(90° - 52°)}$

$= 1 + \dfrac{\tan 38°}{\tan 38°}$

$= \underline{\underline{2}}$

14. $\tan 25° - \dfrac{1}{\tan 65°} = \tan(90° - 65°) - \dfrac{1}{\tan 65°}$

$= \dfrac{1}{\tan 65°} - \dfrac{1}{\tan 65°}$

$= \underline{\underline{0}}$

Level 2

15.
$$\cos 43° = \sin(75° - \theta)$$
$$\sin(90° - 43°) = \sin(75° - \theta)$$
$$\sin 47° = \sin(75° - \theta)$$
$$\therefore \quad 47° = 75° - \theta$$
$$\theta = \underline{\underline{28°}}$$

16.
$$\sin 32° = \cos\left(66° - \frac{\theta}{2}\right)$$
$$\cos(90° - 32°) = \cos\left(66° - \frac{\theta}{2}\right)$$
$$\cos 58° = \cos\left(66° - \frac{\theta}{2}\right)$$
$$\therefore \quad 58° = 66° - \frac{\theta}{2}$$
$$\frac{\theta}{2} = 8°$$
$$\theta = \underline{\underline{16°}}$$

17.
$$\tan(\theta + 15°) = \frac{1}{\tan(\theta + 60°)}$$
$$= \tan[90° - (\theta + 60°)]$$
$$= \tan(30° - \theta)$$
$$\therefore \quad \theta + 15° = 30° - \theta$$
$$2\theta = 15°$$
$$\theta = \underline{\underline{7.5°}}$$

18.
$$\tan(\theta + 45°) = \frac{1}{\tan\frac{\theta}{4}}$$
$$= \tan\left(90° - \frac{\theta}{4}\right)$$
$$\therefore \quad \theta + 45° = 90° - \frac{\theta}{4}$$
$$\frac{5}{4}\theta = 45°$$
$$\theta = \underline{\underline{36°}}$$

19.
$$\frac{\cos^2(90° - \theta)}{1 - \cos\theta} - \sin(90° - \theta) = \frac{\sin^2\theta}{1 - \cos\theta} - \cos\theta$$
$$= \frac{\sin^2\theta - \cos\theta(1 - \cos\theta)}{1 - \cos\theta}$$
$$= \frac{\sin^2\theta + \cos^2\theta - \cos\theta}{1 - \cos\theta}$$
$$= \frac{1 - \cos\theta}{1 - \cos\theta}$$
$$= \underline{\underline{1}}$$

20.
$$\cos^2\theta[1 + \tan^2(90° - \theta)]$$
$$= \cos^2\theta\left(1 + \frac{1}{\tan^2\theta}\right)$$
$$= \cos^2\theta\left(1 + \frac{\cos^2\theta}{\sin^2\theta}\right)$$
$$= \cos^2\theta\left(\frac{\sin^2\theta + \cos^2\theta}{\sin^2\theta}\right)$$
$$= \frac{\cos^2\theta}{\sin^2\theta}$$
$$= \underline{\underline{\frac{1}{\tan^2\theta}}}$$

21.
$$\frac{\cos^2(90° - \theta)}{\tan(90° - \theta)} \times \frac{2}{\sin^3(90° - \theta)\tan^3\theta}$$
$$= \sin^2\theta\tan\theta \times \frac{2}{\cos^3\theta\tan^3\theta}$$
$$= \frac{2\sin^2\theta}{\cos^3\theta\tan^2\theta}$$
$$= \frac{2\sin^2\theta}{\cos^3\theta \times \frac{\sin^2\theta}{\cos^2\theta}}$$
$$= \underline{\underline{\frac{2}{\cos\theta}}}$$

22.
$$\frac{1}{\cos(90° - \theta)} - \frac{\cos(90° - \theta)\tan(90° - \theta)}{\tan\theta}$$
$$= \frac{1}{\sin\theta} - \frac{\sin\theta}{\tan^2\theta}$$
$$= \frac{1}{\sin\theta} - \frac{\sin\theta}{\frac{\sin^2\theta}{\cos^2\theta}}$$
$$= \frac{1}{\sin\theta} - \frac{\cos^2\theta}{\sin\theta}$$
$$= \frac{1 - \cos^2\theta}{\sin\theta}$$
$$= \frac{\sin^2\theta}{\sin\theta}$$
$$= \underline{\underline{\sin\theta}}$$

23.
$$\sin^2 36° + \sin^2 54° = \cos^2(90° - 36°) + \sin^2 54°$$
$$= \cos^2 54° + \sin^2 54°$$
$$= \underline{\underline{1}}$$

24.
$$\sin 12° \cos 78° + \sin 78° \cos 12°$$
$$= \sin 12° \sin(90° - 78°) + \cos(90° - 78°)\cos 12°$$
$$= \sin^2 12° + \cos^2 12°$$
$$= \underline{\underline{1}}$$

25.
$$\sin 15° \tan 75° - \cos 15° = \sin 15° \times \frac{1}{\tan(90° - 75°)} - \cos 15°$$
$$= \sin 15° \times \frac{\cos 15°}{\sin 15°} - \cos 15°$$
$$= \cos 15° - \cos 15°$$
$$= \underline{\underline{0}}$$

26. $\dfrac{\cos 52°}{\tan 38°} - \sin 38° \tan 52°$

$= \cos 52° \tan (90° - 38°) - \cos (90° - 38°) \tan 52°$

$= \cos 52° \tan 52° - \cos 52° \tan 52°$

$= \underline{\underline{0}}$

27. $\sin (90° - \theta) = \cos \theta$

$= \sqrt{1 - \sin^2 \theta} \quad (\because \quad \cos^2 \theta = 1 - \sin^2 \theta)$

$= \sqrt{1 - 0.8^2}$

$= \underline{\underline{0.6}}$

28. $\because \quad \sin (90° - \theta) = \dfrac{3}{4}$

$\therefore \quad \cos \theta = \dfrac{3}{4}$

$\tan \theta = \dfrac{\sin \theta}{\cos \theta}$

$= \dfrac{\sqrt{1 - \cos^2 \theta}}{\cos \theta}$

$= \dfrac{\sqrt{1 - \left(\frac{3}{4}\right)^2}}{\frac{3}{4}}$

$= \sqrt{\dfrac{7}{16}} \times \dfrac{4}{3}$

$= \dfrac{\sqrt{7}}{4} \times \dfrac{4}{3}$

$= \underline{\underline{\dfrac{\sqrt{7}}{3}}}$

29. $\sin^2 (90° - \theta) = \cos^2 \theta$

$= \left(\dfrac{1}{3}\right)^2$

$= \underline{\underline{\dfrac{1}{9}}}$

$\cos^2 (90° - \theta) = \sin^2 \theta$

$= 1 - \cos^2 \theta$

$= 1 - \dfrac{1}{9}$

$= \underline{\underline{\dfrac{8}{9}}}$

$\tan^2 (90° - \theta) = \dfrac{1}{\tan^2 \theta}$

$= \dfrac{\cos^2 \theta}{\sin^2 \theta}$

$= \dfrac{\frac{1}{9}}{\frac{8}{9}}$

$= \underline{\underline{\dfrac{1}{8}}}$

30. $\tan (90° - \theta) = \dfrac{1}{2}$

$\tan \theta = 2 \qquad$ .....(1)

$\dfrac{\sin \theta}{\cos \theta} = 2$

$\sin \theta = 2 \cos \theta \qquad$ .....(2)

$\sin^2 \theta = 4 \cos^2 \theta$

$1 - \cos^2 \theta = 4 \cos^2 \theta$

$\cos^2 \theta = \dfrac{1}{5}$

$\cos \theta = \dfrac{1}{\sqrt{5}} \qquad$ .....(3)

$\dfrac{\sin (90° - \theta)}{1 - \tan \theta} + 2 \cos (90° - \theta) = \dfrac{\cos \theta}{1 - 2} + 2 \sin \theta \qquad$ (by (1))

$= -\cos \theta + 4 \cos \theta \qquad$ (by (2))

$= 3 \cos \theta$

$= \underline{\underline{\dfrac{3}{\sqrt{5}}}} \left(\text{or } \dfrac{3\sqrt{5}}{5}\right) \qquad$ (by (3))

Exercise 9E (p. 9.29)
Level 1

1. L.H.S. $= \dfrac{\tan \theta \cos \theta}{\sin \theta}$

$= \dfrac{\frac{\sin \theta}{\cos \theta} \cos \theta}{\sin \theta}$

$= \dfrac{\sin \theta}{\sin \theta}$

$= 1$

R.H.S. = 1

$\because$ L.H.S. = R.H.S.

$\therefore \quad \dfrac{\tan \theta \cos \theta}{\sin \theta} \equiv 1$

2. R.H.S. $= \tan (90° - \theta) \sin \theta$

$= \dfrac{1}{\tan \theta} \sin \theta$

$= \dfrac{\cos \theta}{\sin \theta} \sin \theta$

$= \cos \theta$

L.H.S. = $\cos \theta$

$\because$ L.H.S. = R.H.S.

$\therefore \quad \cos \theta \equiv \tan (90° - \theta) \sin \theta$

3. R.H.S. $= \dfrac{\sin \theta}{\tan \theta} + \cos \theta$

$= \dfrac{\sin \theta}{\frac{\sin \theta}{\cos \theta}} + \cos \theta$

$= \cos \theta + \cos \theta$

$= 2 \cos \theta$

L.H.S. = $2\cos\theta$

$\because$ L.H.S. = R.H.S.

$\therefore \quad 2 \cos \theta \equiv \dfrac{\sin \theta}{\tan \theta} + \cos \theta$

4. R.H.S. $= \dfrac{1-\cos^2\theta}{\cos(90°-\theta)}$

$= \dfrac{\sin^2\theta}{\sin\theta}$

$= \sin\theta$

L.H.S. $= \sin\theta$

$\because$ L.H.S. = R.H.S.

$\therefore \quad \sin\theta \equiv \dfrac{1-\cos^2\theta}{\cos(90°-\theta)}$

5. L.H.S. $= \sin^3\theta + \sin\theta\cos^2\theta$

$= \sin\theta(\sin^2\theta + \cos^2\theta)$

$= \sin\theta\,(1)$

$= \sin\theta$

R.H.S. $= \sin\theta$

$\because$ L.H.S. = R.H.S.

$\therefore \quad \sin^3\theta + \sin\theta\cos^2\theta \equiv \sin\theta$

6. L.H.S. $= \sin^2\theta - \sin^2(90°-\theta)$

$= \sin^2\theta - \cos^2\theta$

$= \sin^2\theta - (1-\sin^2\theta)$

$= \sin^2\theta - 1 + \sin^2\theta$

$= 2\sin^2\theta - 1$

R.H.S. $= 2\sin^2\theta - 1$

$\because$ L.H.S. = R.H.S.

$\therefore \quad \sin^2\theta - \sin^2(90°-\theta) \equiv 2\sin^2\theta - 1$

7. L.H.S. $= (1+\cos\theta)(1-\cos\theta)$

$= 1-\cos^2\theta$

$= \sin^2\theta$

R.H.S. $= \sin^2\theta$

$\because$ L.H.S. = R.H.S.

$\therefore \quad (1+\cos\theta)(1-\cos\theta) \equiv \sin^2\theta$

8. L.H.S. $= (\sin\theta - \cos\theta)^2$

$= \sin^2\theta - 2\sin\theta\cos\theta + \cos^2\theta$

$= 1 - 2\sin\theta\cos\theta$

R.H.S. $= 1 - 2\sin\theta\cos\theta$

$\because$ L.H.S. = R.H.S.

$\therefore \quad (\sin\theta - \cos\theta)^2 \equiv 1 - 2\sin\theta\cos\theta$

Level 2

9. L.H.S. $= 1 + \tan^2\theta$

$= 1 + \dfrac{\sin^2\theta}{\cos^2\theta}$

$= \dfrac{\cos^2\theta + \sin^2\theta}{\cos^2\theta}$

$= \dfrac{1}{\cos^2\theta}$

R.H.S. $= \dfrac{1}{\cos^2\theta}$

$\because$ L.H.S. = R.H.S.

$\therefore \quad 1 + \tan^2\theta \equiv \dfrac{1}{\cos^2\theta}$

10. L.H.S. $= \cos^2\theta\tan^2\theta$

$= \cos^2\theta \times \dfrac{\sin^2\theta}{\cos^2\theta}$

$= \sin^2\theta$

R.H.S. $= 1 - \sin^2(90°-\theta)$

$= 1 - \cos^2\theta$

$= \sin^2\theta$

$\because$ L.H.S. = R.H.S.

$\therefore \quad \cos^2\theta\tan^2\theta \equiv 1 - \sin^2(90°-\theta)$

11. L.H.S. $= \sin\theta + \dfrac{\cos\theta}{\tan\theta}$

$= \sin\theta + \dfrac{\cos\theta}{\dfrac{\sin\theta}{\cos\theta}}$

$= \sin\theta + \dfrac{\cos^2\theta}{\sin\theta}$

$= \dfrac{\sin^2\theta + \cos^2\theta}{\sin\theta}$

$= \dfrac{1}{\sin\theta}$

R.H.S. $= \dfrac{1}{\sin\theta}$

$\because$ L.H.S. = R.H.S.

$\therefore \quad \sin\theta + \dfrac{\cos\theta}{\tan\theta} \equiv \dfrac{1}{\sin\theta}$

12. L.H.S. $= \dfrac{1}{\sin\theta\cos\theta} - \tan\theta$

$= \dfrac{1}{\sin\theta\cos\theta} - \dfrac{\sin\theta}{\cos\theta}$

$= \dfrac{1}{\sin\theta\cos\theta} - \dfrac{\sin^2\theta}{\sin\theta\cos\theta}$

$= \dfrac{\cos^2\theta}{\sin\theta\cos\theta}$

$= \dfrac{\cos\theta}{\sin\theta}$

$= \dfrac{1}{\tan\theta}$

R.H.S. $= \tan(90°-\theta)$

$= \dfrac{1}{\tan\theta}$

$\because$ L.H.S. = R.H.S.

$\therefore \quad \dfrac{1}{\sin\theta\cos\theta} - \tan\theta \equiv \tan(90°-\theta)$

13. L.H.S. $= \tan^2\theta\sin^2\theta$

$= \tan^2\theta(1-\cos^2\theta)$

$= \tan^2\theta - \tan^2\theta\cos^2\theta$

$= \tan^2\theta - \dfrac{\sin^2\theta}{\cos^2\theta}\cos^2\theta$

$= \tan^2\theta - \sin^2\theta$

R.H.S. $= \tan^2\theta - \sin^2\theta$

$\because$ L.H.S. = R.H.S.

$\therefore \quad \tan^2\theta\sin^2\theta \equiv \tan^2\theta - \sin^2\theta$

14. L.H.S. $= \dfrac{\tan(90° - \theta)}{1 + \tan^2(90° - \theta)}$

$= \dfrac{1}{\tan\theta\left(1 + \dfrac{1}{\tan^2\theta}\right)}$

$= \dfrac{1}{\tan\theta + \dfrac{1}{\tan\theta}}$

$= \dfrac{1}{\dfrac{\sin\theta}{\cos\theta} + \dfrac{\cos\theta}{\sin\theta}}$

$= \dfrac{1}{\dfrac{\sin^2\theta + \cos^2\theta}{\sin\theta\cos\theta}}$

$= \dfrac{1}{\dfrac{1}{\sin\theta\cos\theta}}$

$= \sin\theta\cos\theta$

R.H.S. $= \sin\theta\cos\theta$

$\because$ L.H.S. = R.H.S.

$\therefore$ $\dfrac{\tan(90° - \theta)}{1 + \tan^2(90° - \theta)} \equiv \sin\theta\cos\theta$

15. **(a)** L.H.S. $= \sin^2\theta - \cos^2\theta$

$= (\sin\theta - \cos\theta)(\sin\theta + \cos\theta)$

R.H.S. $= \sin\theta - \cos\theta$

$\because$ L.H.S. $\neq$ R.H.S.

$\therefore$ $\sin^2\theta - \cos^2\theta = \sin\theta - \cos\theta$ is not an identity.

(b) L.H.S. $= \sin^4\theta - \cos^4\theta$

$= (\sin^2\theta - \cos^2\theta)(\sin^2\theta + \cos^2\theta)$

$= (\sin^2\theta - \cos^2\theta)(1)$

$= \sin^2\theta - \cos^2\theta$

R.H.S. $= \sin^2\theta - \cos^2\theta$

$\because$ L.H.S. = R.H.S.

$\therefore$ $\sin^4\theta - \cos^4\theta = \sin^2\theta - \cos^2\theta$ is an identity.

Revision Exercise 9 (p. 9.33)
Level 1

1. $\cos 60° + 2\tan 45° = \dfrac{1}{2} + 2 \times 1$

$= \underline{\underline{\dfrac{5}{2}}}$

2. $5\cos 45° \tan^2 60° - 3\sin 45° = 5 \times \dfrac{\sqrt{2}}{2} \times (\sqrt{3})^2 - 3 \times \dfrac{\sqrt{2}}{2}$

$= \dfrac{15\sqrt{2}}{2} - \dfrac{3\sqrt{2}}{2}$

$= \dfrac{12\sqrt{2}}{2}$

$= \underline{\underline{6\sqrt{2}}}$

3. $\sin^2 30° + \cos^2 30° + \tan^2 30° = \left(\dfrac{1}{2}\right)^2 + \left(\dfrac{\sqrt{3}}{2}\right)^2 + \left(\dfrac{1}{\sqrt{3}}\right)^2$

$= \dfrac{1}{4} + \dfrac{3}{4} + \dfrac{1}{3}$

$= \underline{\underline{\dfrac{4}{3}}}$

4. $\dfrac{2 - \sin^2 30° \cos 60°}{\cos^2 45°} = \dfrac{2 - \left(\dfrac{1}{2}\right)^2 \times \dfrac{1}{2}}{\left(\dfrac{1}{\sqrt{2}}\right)^2}$

$= \dfrac{2 - \dfrac{1}{8}}{\dfrac{1}{2}}$

$= \dfrac{15}{8} \times 2$

$= \underline{\underline{\dfrac{15}{4}}}$

5. $\sin\theta = \dfrac{\tan 45°}{\sin 45° + \cos 45°}$

$= \dfrac{1}{\dfrac{1}{\sqrt{2}} + \dfrac{1}{\sqrt{2}}}$

$= \dfrac{1}{\dfrac{2}{\sqrt{2}}}$

$= \dfrac{\sqrt{2}}{2}$

$\therefore$ $\theta = \underline{\underline{45°}}$

6. $\tan\theta = \dfrac{\cos 60°}{\sin 30° \tan 30°}$

$= \dfrac{\dfrac{1}{2}}{\dfrac{1}{2} \times \dfrac{1}{\sqrt{3}}}$

$= \sqrt{3}$

$\therefore$ $\theta = \underline{\underline{60°}}$

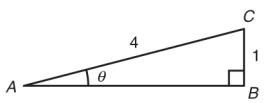
7. Construct $\triangle ABC$ as shown with $\sin\theta = \dfrac{1}{4}$.

By Pythagoras' theorem,

$AB = \sqrt{AC^2 - BC^2}$

$= \sqrt{4^2 - 1^2}$

$= \sqrt{15}$



By definition,

$\cos\theta = \dfrac{AB}{AC} = \underline{\underline{\dfrac{\sqrt{15}}{4}}}$

$\tan\theta = \dfrac{BC}{AB} = \underline{\underline{\dfrac{1}{\sqrt{15}}}} \left(\text{or } \dfrac{\sqrt{15}}{15}\right)$

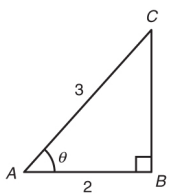
8. Construct $\triangle ABC$ as shown with $\cos\theta = \dfrac{2}{3}$.

By Pythagoras' theorem,

$BC = \sqrt{AC^2 - AB^2}$

$= \sqrt{3^2 - 2^2}$

$= \sqrt{5}$



By definition,

$$\sin\theta = \frac{BC}{AC} = \underline{\underline{\frac{\sqrt{5}}{3}}}$$

$$\tan\theta = \frac{BC}{AB} = \underline{\underline{\frac{\sqrt{5}}{2}}}$$

9. $\tan\theta = 2.5$

$$= \frac{25}{10}$$

$$= \frac{5}{2}$$

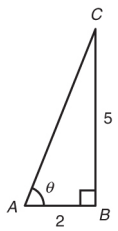
Construct $\triangle ABC$ as shown with $\tan\theta = \frac{5}{2}$.

By Pythagoras' theorem,

$$AC = \sqrt{AB^2 + BC^2}$$

$$= \sqrt{2^2 + 5^2}$$

$$= \sqrt{29}$$



By definition,

$$\sin\theta = \frac{BC}{AC} = \underline{\underline{\frac{5}{\sqrt{29}}}} \left(\text{or } \frac{5\sqrt{29}}{29}\right)$$

$$\cos\theta = \frac{AB}{AC} = \underline{\underline{\frac{2}{\sqrt{29}}}} \left(\text{or } \frac{2\sqrt{29}}{29}\right)$$

10. $2\sin^2 46° + 2\sin^2 44° = 2[\cos^2(90° - 46°) + \sin^2 44°]$

$$= 2(\cos^2 44° + \sin^2 44°)$$

$$= 2(1)$$

$$= \underline{\underline{2}}$$

11. $\cos^2 32° \tan^2 32° - \cos^2 58°$

$$= \cos^2 32° \times \frac{\sin^2 32°}{\cos^2 32°} - \sin^2(90° - 58°)$$

$$= \sin^2 32° - \sin^2 32°$$

$$= \underline{\underline{0}}$$

12. $\dfrac{\tan 11° \tan 79°}{1 + 2\tan 11° \tan 79°} = \dfrac{\tan(90° - 79°)\tan 79°}{1 + 2\tan(90° - 79°)\tan 79°}$

$$= \frac{\frac{1}{\tan 79°} \times \tan 79°}{1 + 2 \times \frac{1}{\tan 79°} \times \tan 79°}$$

$$= \frac{1}{1+2}$$

$$= \underline{\underline{\frac{1}{3}}}$$

13. $\dfrac{1}{\tan^2\theta} - \dfrac{1}{\sin^2\theta} = \dfrac{\cos^2\theta}{\sin^2\theta} - \dfrac{1}{\sin^2\theta}$

$$= \frac{\cos^2\theta - 1}{\sin^2\theta}$$

$$= \frac{-\sin^2\theta}{\sin^2\theta}$$

$$= \underline{\underline{-1}}$$

14. $\dfrac{\sin^3\theta - \sin\theta}{\cos^3\theta} = \dfrac{\sin\theta(\sin^2\theta - 1)}{\cos^3\theta}$

$$= \frac{\sin\theta(-\cos^2\theta)}{\cos^3\theta}$$

$$= -\frac{\sin\theta}{\cos\theta}$$

$$= \underline{\underline{-\tan\theta}}$$

15. $\dfrac{1+\cos\theta}{\sin^2\theta} - \dfrac{1}{1-\cos\theta} = \dfrac{1+\cos\theta}{1-\cos^2\theta} - \dfrac{1}{1-\cos\theta}$

$$= \frac{1+\cos\theta}{(1+\cos\theta)(1-\cos\theta)} - \frac{1}{1-\cos\theta}$$

$$= \frac{1}{1-\cos\theta} - \frac{1}{1-\cos\theta}$$

$$= \underline{\underline{0}}$$

16. $\left(\dfrac{\tan\theta}{\sin\theta} - \cos\theta\right)\left(\dfrac{1}{\sin\theta} - \sin\theta\right)$

$$= \left(\frac{\frac{\sin\theta}{\cos\theta}}{\sin\theta} - \cos\theta\right)\left(\frac{1-\sin^2\theta}{\sin\theta}\right)$$

$$= \left(\frac{1}{\cos\theta} - \cos\theta\right)\frac{\cos^2\theta}{\sin\theta}$$

$$= \frac{1-\cos^2\theta}{\cos\theta} \times \frac{\cos^2\theta}{\sin\theta}$$

$$= \frac{\sin^2\theta\cos\theta}{\sin\theta}$$

$$= \underline{\underline{\sin\theta\cos\theta}}$$

17. $4\cos^2(90° - \theta) + 4\sin^2(90° - \theta) + 4$

$$= 4\sin^2\theta + 4\cos^2\theta + 4$$

$$= 4(\sin^2\theta + \cos^2\theta) + 4$$

$$= 4(1) + 4$$

$$= \underline{\underline{8}}$$

18. $\sin^2(90° - \theta) + \dfrac{\cos^2\theta}{\tan^2(90° - \theta)} = \cos^2\theta + \dfrac{\cos^2\theta}{\frac{\cos^2\theta}{\sin^2\theta}}$

$$= \cos^2\theta + \sin^2\theta$$

$$= \underline{\underline{1}}$$

19. $\dfrac{1}{\sin^2(90° - \theta)} - 1 = \dfrac{1}{\cos^2\theta} - 1$

$$= \frac{1-\cos^2\theta}{\cos^2\theta}$$

$$= \frac{\sin^2\theta}{\cos^2\theta}$$

$$= \underline{\underline{\tan^2\theta}}$$

20. $\sin(90°-\theta)\sin\theta\tan(90°-\theta)+\sin^2\theta$

$=\cos\theta\sin\theta\dfrac{1}{\tan\theta}+\sin^2\theta$

$=\cos\theta\sin\theta\times\dfrac{\cos\theta}{\sin\theta}+\sin^2\theta$

$=\cos^2\theta+\sin^2\theta$

$=\underline{\underline{1}}$

21. $\sin\theta=\cos(\theta-30°)$

$\cos(90°-\theta)=\cos(\theta-30°)$

$\therefore\quad 90°-\theta=\theta-30°$

$2\theta=120°$

$\theta=\underline{\underline{60°}}$

22. $\cos(\theta-45°)=\sin 65°$

$=\cos(90°-65°)$

$\therefore\quad \theta-45°=90°-65°$

$\theta=\underline{\underline{70°}}$

23. $\tan 28°=\dfrac{1}{\tan(\theta-24°)}$

$=\tan[90°-(\theta-24°)]$

$\therefore\quad 28°=90°-\theta+24°$

$\theta=\underline{\underline{86°}}$

24. L.H.S. $=\dfrac{(1-\cos\theta)(1+\cos\theta)}{\cos^2(90°-\theta)}$

$=\dfrac{1-\cos^2\theta}{\sin^2\theta}$

$=\dfrac{\sin^2\theta}{\sin^2\theta}$

$=1$

R.H.S. = 1

$\because$ L.H.S. = R.H.S.

$\therefore\quad \dfrac{(1-\cos\theta)(1+\cos\theta)}{\cos^2(90°-\theta)}\equiv 1$

25. L.H.S. $=1-\sin^2\theta$

$=\cos^2\theta$

R.H.S. $=\sin\theta\tan(90°-\theta)\cos\theta$

$=\sin\theta\times\dfrac{\cos\theta}{\sin\theta}\times\cos\theta$

$=\cos^2\theta$

$\because$ L.H.S. = R.H.S.

$\therefore\quad 1-\sin^2\theta\equiv\sin\theta\tan(90°-\theta)\cos\theta$

26. (a) Consider $\triangle ABD$.

$\cos 45°=\dfrac{2\text{ cm}}{BD}$

$BD=\dfrac{2}{\frac{1}{\sqrt{2}}}\text{ cm}$

$=\underline{\underline{2\sqrt{2}\text{ cm}}}$

(b) Consider $\triangle ABD$.

$\tan 45°=\dfrac{AD}{2\text{ cm}}$

$AD=2(1)\text{ cm}$

$=2\text{ cm}$

Consider $\triangle BCD$.

$\cos 45°=\dfrac{BD}{BC}$

$\dfrac{1}{\sqrt{2}}=\dfrac{2\sqrt{2}\text{ cm}}{BC}$

$BC=4\text{cm}$

Area of $ABCD=\dfrac{1}{2}\times(AD+BC)\times AB$

$=\dfrac{1}{2}\times(2+4)\times 2\text{ cm}^2$

$=\underline{\underline{6\text{cm}^2}}$

27. (a) $\tan 30°=\dfrac{CD}{AB}$

$\dfrac{\sqrt{3}}{3}=\dfrac{CD}{3\text{ m}}$

$CD=\underline{\underline{\sqrt{3}\text{ m}}}$

(b) $BC=BD+CD$

$=\underline{\underline{(1.5+\sqrt{3})\text{ m}}}\left(\text{or }\dfrac{3+2\sqrt{3}}{2}\text{ m}\right)$

28. (a) Consider $\triangle ACD$.

$\tan\theta=\dfrac{3}{3}$

$=1$

$\theta=45°$

(b) Consider $\triangle BAE$.

$\sin 45°=\dfrac{BE}{AB}$

$\dfrac{1}{\sqrt{2}}=\dfrac{BE}{5\text{ m}}$

$BE=\dfrac{5}{\sqrt{2}}\text{ m}\left(\text{or }\dfrac{5\sqrt{2}}{2}\text{ m}\right)$

$\therefore$ The vertical distance from B to the ground is $\dfrac{5}{\sqrt{2}}\text{ m}\left(\text{or }\dfrac{5\sqrt{2}}{2}\text{ m}\right)$.

Level 2

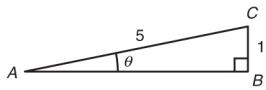
29. (a) Construct $\triangle ABC$ as shown with $\sin\theta=\dfrac{1}{5}$.

By Pythagoras' theorem,

$AB=\sqrt{AC^2-BC^2}$

$=\sqrt{5^2-1^2}$

$=\sqrt{24}$



By definition,

$\cos\theta=\dfrac{AB}{AC}=\underline{\underline{\dfrac{\sqrt{24}}{5}}}\left(\text{or }\dfrac{2\sqrt{6}}{5}\right)$

$\tan\theta=\dfrac{BC}{AB}=\underline{\underline{\dfrac{1}{\sqrt{24}}}}\left(\text{or }\dfrac{\sqrt{6}}{12}\right)$

(b) $\dfrac{2}{\sin^2(90°-\theta)[1+\tan^2(90°-\theta)]}$

$= \dfrac{2}{\cos^2\theta\left(1+\dfrac{1}{\tan^2\theta}\right)}$

$= \dfrac{2}{\left(\dfrac{\sqrt{24}}{5}\right)^2[1+(\sqrt{24})^2]}$

$= \dfrac{2}{\dfrac{24}{25}\times 25}$

$= \underline{\underline{\dfrac{1}{12}}}$

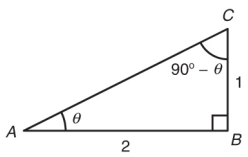
30. (a) Construct $\triangle ABC$ as shown with $\tan(90°-\theta) = \dfrac{2}{1}$.

By Pythagoras' theorem,

$AC = \sqrt{AB^2 + BC^2}$

$= \sqrt{2^2+1^2}$

$= \sqrt{5}$



By definition,

$\sin\theta = \dfrac{BC}{AC} = \underline{\underline{\dfrac{1}{\sqrt{5}}}}\left(\text{or } \dfrac{\sqrt{5}}{5}\right)$

$\cos\theta = \dfrac{AB}{AC} = \underline{\underline{\dfrac{2}{\sqrt{5}}}}\left(\text{or } \dfrac{2\sqrt{5}}{5}\right)$

(b) $\dfrac{2\sin\theta\cos\theta}{1+\cos^2\theta-\sin^2\theta} = \dfrac{2\sin\theta\cos\theta}{1-\sin^2\theta+\cos^2\theta}$

$= \dfrac{2\sin\theta\cos\theta}{2\cos^2\theta}$

$= \dfrac{\sin\theta}{\cos\theta}$

$= \tan\theta$

$= \underline{\underline{\dfrac{1}{2}}}$

31. (a) L.H.S. $= \dfrac{\sin\theta+\sin(90°-\theta)}{1+\tan(90°-\theta)}$

$= \dfrac{\sin\theta+\cos\theta}{1+\dfrac{\cos\theta}{\sin\theta}}$

$= \dfrac{\sin\theta+\cos\theta}{\dfrac{\sin\theta+\cos\theta}{\sin\theta}}$

$= \sin\theta$

R.H.S. $= \cos(90°-\theta)$

$= \sin\theta$

$\because$ L.H.S. = R.H.S.

$\therefore \dfrac{\sin\theta+\sin(90°-\theta)}{1+\tan(90°-\theta)} \equiv \cos(90°-\theta)$

(b) $\dfrac{\sin\theta+\sin(90°-\theta)}{1+\tan(90°-\theta)} = \cos 30°$

$\cos(90°-\theta) = \cos 30°$ (from (a))

$\therefore$ $90°-\theta = 30°$

$\theta = \underline{\underline{60°}}$

32. (a) $\cos(42°-\theta)\tan(42°-\theta) = \cos(2\theta+18°)$

$\cos(42°-\theta)\times\dfrac{\sin(42°-\theta)}{\cos(42°-\theta)} = \cos(2\theta+18°)$

$\therefore$ $\sin(42°-\theta) = \sin[90°-(2\theta+18°)]$

$42°-\theta = 90°-2\theta-18°$

$\theta = \underline{\underline{30°}}$

(b) $3\cos\theta\sin(15°+\theta)\tan 2\theta = 3\cos 30°\sin 45°\tan 60°$

$= 3\times\dfrac{\sqrt{3}}{2}\times\dfrac{1}{\sqrt{2}}\times\sqrt{3}$

$= \underline{\underline{\dfrac{9}{2\sqrt{2}}}}\left(\text{or } \dfrac{9\sqrt{2}}{4}\right)$

33. (a) L.H.S. $= \dfrac{1-\cos\theta}{1+\cos\theta}$

$= \dfrac{(1-\cos\theta)(1-\cos\theta)}{(1+\cos\theta)(1-\cos\theta)}$

$= \dfrac{(1-\cos\theta)^2}{1-\cos^2\theta}$

$= \left(\dfrac{1-\cos\theta}{\sin\theta}\right)^2$

R.H.S. $= \left(\dfrac{1-\cos\theta}{\sin\theta}\right)^2$

$\because$ L.H.S. = R.H.S.

$\therefore \dfrac{1-\cos\theta}{1+\cos\theta} \equiv \left(\dfrac{1-\cos\theta}{\sin\theta}\right)^2$

(b) L.H.S. $= \dfrac{1-\cos\theta}{1+\cos\theta}$

$= \left(\dfrac{1-\cos\theta}{\sin\theta}\right)^2$

R.H.S. $= \left(\dfrac{1}{\sin\theta}-\dfrac{1}{\tan\theta}\right)^2$

$= \left(\dfrac{1}{\sin\theta}-\dfrac{\cos\theta}{\sin\theta}\right)^2$

$= \left(\dfrac{1-\cos\theta}{\sin\theta}\right)^2$

$\because$ L.H.S. = R.H.S.

$\therefore \dfrac{1-\cos\theta}{1+\cos\theta} \equiv \left(\dfrac{1}{\sin\theta}-\dfrac{1}{\tan\theta}\right)^2$

34. Consider $\triangle ABD$.

$\angle BDC = \angle A + \angle DBA$ (ext. $\angle$ of $\triangle$)

$= 30° + 30°$

$= 60°$

In $\triangle BAC$,

$\tan 30° = \dfrac{\sqrt{3}\text{ cm}}{AC}$

$AC = \dfrac{\sqrt{3}}{\tan 30°}\text{ cm}$

$= \dfrac{\sqrt{3}}{\dfrac{1}{\sqrt{3}}}\text{ cm}$

$= 3\text{ cm}$

In $\triangle BDC$,

$$\tan\angle BDC = \frac{\sqrt{3}\text{ cm}}{DC}$$

$$DC = \frac{\sqrt{3}}{\tan 60^\circ}\text{ cm}$$

$$= \frac{\sqrt{3}}{\sqrt{3}}\text{ cm}$$

$$= 1\text{ cm}$$

$\therefore$ Area of $\triangle ABD = \frac{1}{2}\times AD\times BC$

$$= \frac{1}{2}\times(AC - DC)\times BC$$

$$= \frac{1}{2}\times(3-1)\times\sqrt{3}\text{ cm}^2$$

$$= \underline{\underline{\sqrt{3}\text{ cm}^2}}$$

35. (a) In $\triangle ECD$,

$$\cos 30^\circ = \frac{EC}{2\text{ cm}}$$

$$EC = 2\cos 30^\circ\text{ cm}$$

$$= 2\times\frac{\sqrt{3}}{2}\text{ cm}$$

$$= \sqrt{3}\text{ cm}$$

In $\triangle ACB$,

$$\cos 30^\circ = \frac{BC}{4\text{ cm}}$$

$$BC = 4\cos 30^\circ\text{ cm}$$

$$= 4\times\frac{\sqrt{3}}{2}\text{ cm}$$

$$= 2\sqrt{3}\text{ cm}$$

$\therefore$ $BE = BC - EC$

$$= (2\sqrt{3} - \sqrt{3})\text{ cm}$$

$$= \underline{\underline{\sqrt{3}\text{ cm}}}$$

(b) In $\triangle ECD$,

$$\sin 30^\circ = \frac{ED}{2\text{ cm}}$$

$$ED = 2\sin 30^\circ\text{ cm}$$

$$= 2\times\frac{1}{2}\text{ cm}$$

$$= 1\text{ cm}$$

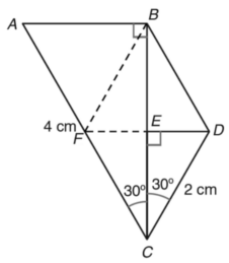
In $\triangle BED$,
by Pythagoras' theorem,

$$BD = \sqrt{BE^2 + ED^2}$$

$$= \sqrt{(\sqrt{3})^2 + 1^2}\text{ cm}$$

$$= \underline{\underline{2\text{ cm}}}$$

(c) With the notations in the figure,
in $\triangle FCE$,



$$\tan 30^\circ = \frac{EF}{EC}$$

$$EF = EC\tan 30^\circ$$

$$= \sqrt{3}\times\frac{1}{\sqrt{3}}\text{ cm}$$

$$= 1\text{ cm}$$

Consider quadrilateral $BFCD$.

$\because$ $EF = ED$, $BE = EC$ and $BC \perp FD$

$\therefore$ $BFCD$ is a rhombus.

36. $\because$ $BCDE$ is a rectangle.

$\therefore$ $BE = CD = y$ and $DE = BC = 6\sqrt{3}$

In $\triangle ABE$,

$$\tan 30^\circ = \frac{AE}{BE}$$

$$AE = y\tan 30^\circ$$

$$= \frac{y}{\sqrt{3}}$$

In $\triangle ACD$,

$AD = AE + ED$

$$x = \frac{y}{\sqrt{3}} + 6\sqrt{3}\text{.....(1)}$$

$$\tan 60^\circ = \frac{x}{y}$$

$$x = y\tan 60^\circ$$

$$x = y\sqrt{3}\text{.....(2)}$$

By substituting (2) into (1), we have

$$y\sqrt{3} = \frac{y}{\sqrt{3}} + 6\sqrt{3}$$

$$y\left(\sqrt{3} - \frac{1}{\sqrt{3}}\right) = 6\sqrt{3}$$

$$y(3-1) = 18$$

$$y = \frac{18}{2}$$

$$= \underline{\underline{9}}$$

By substituting $y = 9$ into (2), we have

$$x = \underline{\underline{9\sqrt{3}}}$$

37. (a) Consider $\triangle DBF$.

$$\sin 60^\circ = \frac{FB}{FD}$$

$$\frac{\sqrt{3}}{2} = \frac{FB}{4\text{ m}}$$

$$FB = 2\sqrt{3}\text{ m}$$

Consider $\triangle ABC$.

$$\sin 30^\circ = \frac{CB}{CA}$$

$$\frac{1}{2} = \frac{CB}{4\text{ m}}$$

$$CB = 2\text{ m}$$

$$FC = FB - CB$$

$$= (2\sqrt{3} - 2)\text{ m}$$

$\therefore$ The top of the ladder has been lowered $(2\sqrt{3} - 2)$ m .

(b) Consider $\triangle ABC$.

$$\cos 30^\circ = \frac{AB}{AC}$$

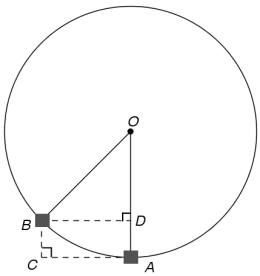
$$\frac{\sqrt{3}}{2} = \frac{AB}{4\text{ m}}$$

$$AB = 2\sqrt{3}\text{ m}$$

Consider $\triangle ABF$.
By Pythagoras' theorem,

$$AF = \sqrt{AB^2 + BF^2}$$
$$= \sqrt{(2\sqrt{3})^2 + (2\sqrt{3})^2} \text{ m}$$
$$= \underline{\underline{\sqrt{24} \text{ m}}} \text{ (or } 2\sqrt{6} \text{ m)}$$

38. Let O be the centre of the ferris wheel.
Suppose the gondola travelled from A to B in 5 s.



Degree of the gondola travelled after 5 s $= \angle BOA$

$$= 360° \times \frac{5}{40}$$
$$= 45°$$

Radius of the ferris wheel $= \frac{1}{2} \times 24 \text{ m}$

$$= 12 \text{ m}$$

Consider $\triangle BOD$.

$$\cos \angle BOD = \frac{OD}{OB}$$
$$OD = OB \cos \angle BOD$$
$$= 12 \cos 45° \text{ m}$$
$$= 12 \times \frac{\sqrt{2}}{2} \text{ m}$$
$$= 6\sqrt{2} \text{ m}$$
$$DA = OA - OD$$
$$= (12 - 6\sqrt{2}) \text{ m}$$
$$BC = DA$$
$$= (12 - 6\sqrt{2}) \text{ m}$$

$\therefore$ He rises $(12 - 6\sqrt{2})$ m after 5 s.

Challenging Questions (p. 9.36)

1. Consider $\triangle BCD$.

$$\sin 45° = \frac{CD}{BC}$$
$$BC = \frac{1}{\frac{1}{\sqrt{2}}}$$
$$= \sqrt{2}$$

Consider $\triangle ABC$.

$$\cos 30° = \frac{BC}{AB}$$
$$AB = \frac{\sqrt{2}}{\frac{\sqrt{3}}{2}}$$
$$= \frac{2\sqrt{2}}{\sqrt{3}}$$
$$= \frac{2\sqrt{6}}{3}$$
$$\sin 30° = \frac{AC}{AB}$$
$$AC = \frac{2\sqrt{6}}{3} \times \frac{1}{2}$$
$$= \frac{\sqrt{6}}{3}$$
$$\angle ECB = \angle CBD = 45° \qquad \text{(alt. } \angle\text{s, } EC \,//\, BD\text{)}$$
$$\angle ACE = 90° - 45°$$
$$= 45°$$

Consider $\triangle ACE$.

$$\sin 45° = \frac{AE}{AC}$$
$$AE = \frac{\sqrt{6}}{3} \times \frac{1}{\sqrt{2}}$$
$$= \frac{\sqrt{3}}{3}$$

Consider $\triangle ABG$.

$$\sin 75° = \frac{AG}{AB}$$
$$= \frac{AE + EG}{AB}$$
$$= \frac{\frac{\sqrt{3}}{3} + 1}{\frac{2\sqrt{6}}{3}}$$
$$= \frac{\sqrt{3} + 3}{2\sqrt{6}}$$
$$= \frac{3\sqrt{2} + 3\sqrt{6}}{12}$$
$$= \underline{\underline{\frac{\sqrt{2} + \sqrt{6}}{4}}}$$

2. Consider the numerator.

$$\tan 1° \tan 2° \tan 3° \ldots \tan 44° \tan 45° \tan 46° \ldots \tan 87° \tan 88° \tan 89°$$
$$= \tan 1° \tan 2° \tan 3° \ldots \tan 44° \tan 45° \left(\frac{1}{\tan 44°}\right) \ldots$$
$$\left(\frac{1}{\tan 3°}\right)\left(\frac{1}{\tan 2°}\right)\left(\frac{1}{\tan 1°}\right)$$
$$= \tan 45°$$
$$= 1$$

Consider the denominator.

$$\cos^2 1° + \cos^2 2° + ... + \cos^2 44° + \cos^2 45° + \cos^2 46° + ...$$
$$+ \cos^2 88° + \cos^2 89°$$
$$= \cos^2 1° + \cos^2 2° + ... + \cos^2 44° + \cos^2 45° + ...$$
$$+ \sin^2(90° - 46°) + \sin^2(90° - 88°) + \sin^2(90° - 89°)$$
$$= \cos^2 1° + \cos^2 2° + ... + \cos^2 44° + \cos^2 45° + \sin^2 44° + ...$$
$$+ \sin^2 2° + \sin^2 1°$$
$$= \underbrace{1 + 1 + ... + 1 + 1}_{44 \text{ terms}} + \cos^2 45°$$
$$= 44 + \frac{1}{2}$$
$$= \frac{89}{2}$$
$$\therefore \frac{\tan 1° \tan 2° \tan 3° ... \tan 87° \tan 88° \tan 89°}{\cos^2 1° + \cos^2 2° + \cos^2 3° + ... + \cos^2 87° + \cos^2 88° + \cos^2 89°}$$
$$= \frac{1}{\left(\frac{89}{2}\right)}$$
$$= \underline{\underline{\frac{2}{89}}}$$